Adaptive methods for finite volume approximations

(thesis summary)

Stan Tomov

The use of *adaptive methods for finite volume methods*, as the use of any adaptive method, has two purposes :

- to get reliable control of the error coming from the discretization of the considered PDE problem, and
- to use the available computational resources efficiently.

The adaptive methods achieve balance between these goals by doing (1) local grid refinement, based on (2) a posteriori error analysis. Additionally, fundamental physical limitations on the computer processing speed may require the exploitation of parallelism, so we will require the local grid refinement to be done in parallel.

The thesis has the following main components, each of which is discussed briefly below :
(1) Introductory part;
(2) A posteriori error estimators;
(3) Parallel mesh refinement;
(4) Numerical results.

I. Introductory part.

This is an introduction to both adaptive and finite volume methods. It includes a general adaptive methods framework, review of the literature for a posteriori error estimates for finite element, numerical motivation of why to use adaptive methods, finite volume methods theory, why and when they are prefered, etc.

The derived in the thesis a posteriori error estimators are for convection-diffusion-reaction problems. The general form of such problems and their finite volume discretization are introduced. For convection-dominated problems the standard approximation techniques give oscillating numerical results and mass disbalance. Different up-wind approximations, that give a desired stabilization of the discretization, are discussed.

II. A posteriori error estimates.

A posteriori error estimators and indicators for the finite element method have been widely used and studied for the past 25 years. On the other hand, there are only few works on a posteriori error estimates for finite volume methods. The goal in this part of the thesis is to

- construct,

- theoretically justify, and
- test

finite volume estimators obtained by exploiting the ideas from the main a posteriori error estimators known from the finite element method. Following the goal, estimators based on (1) local residuals, (2) local Dirichlet or Neumann problems, and (3) Zienciewicz-Zhu type estimators were constructed and theoretically justified by proving (under certain assumptions) that they provide, up to a certain constant, both lower and upper bounds for the error. The dependence of the constants on the shape of the elements and on the problem's parameters is discussed. The analysis is performed in global H^{1}, L^2 and energy norms. The equivalence of the error estimators and the error in the above norms is numerically confirmed for test problems with known solutions. The analysis takes into account the used up-wind scheme.

III. Parallel mesh refinement.

Parallel grid generation tools play an important role in the scientific research that requires the power of high performance parallel computers. Here I explain the algorithms and the implementation issues concerning the creation of such tools. This includes refinement/de-refinement algorithms, techniques to maintain load balance, general issues in developing parallel FE/FV codes based on domain decomposition, ways to maintain mesh conformity on the different refinement levels, data structures. I used the presented techniques to create such a parallel mesh generation tool, named *ParaGrid*. *ParaGrid* is software that takes as input a coarse tetrahedral mesh, which describes well the domain, splits it using METIS, distributes the partitioning among the available processors and generates in parallel a sequence of meshes. It has it's internal solvers and is able to generate various Finite Element/Volume discretizations. The maintained data structures allow *ParaGrid* to be easily connected to (or used to provide data to) external parallel finite element/volume solvers based on domain decomposition.

Different strategies to maintain the load balance are tested. One is to use a posteriori error estimates as weights in an element based initial splitting of the coarse mesh into sub-domains. The idea is that if the error is balanced over the subdomains then the local refinements that follow will produce computational mesh with number of tetrahedrons balanced over the sub-domains. Such strategy doesn't work for time dependent problems, for which case element "migration" between the subdomains is considered.

IV. Numerical results.

This part contains various experiments that numerically illustrate the obtained theoretical results. There are two groups of experiments. The first one is experiments with known exact solutions, for which case the exact error is compared with the numerical results for the different error estimators. The second group is for problems with unknown exact solutions. There are results giving the behavior of the error estimators for problems varying from diffusion to convection dominated. Also, there are results concerning the parallel computations. These are tables for the load balance and the efficiency of the implemented algorithms.

A general conclusion is that the efficiency of the derived error estimators and parallel grid refinement techniques is supported by the numerical experiments.