COSC 462

Parallel Sorting

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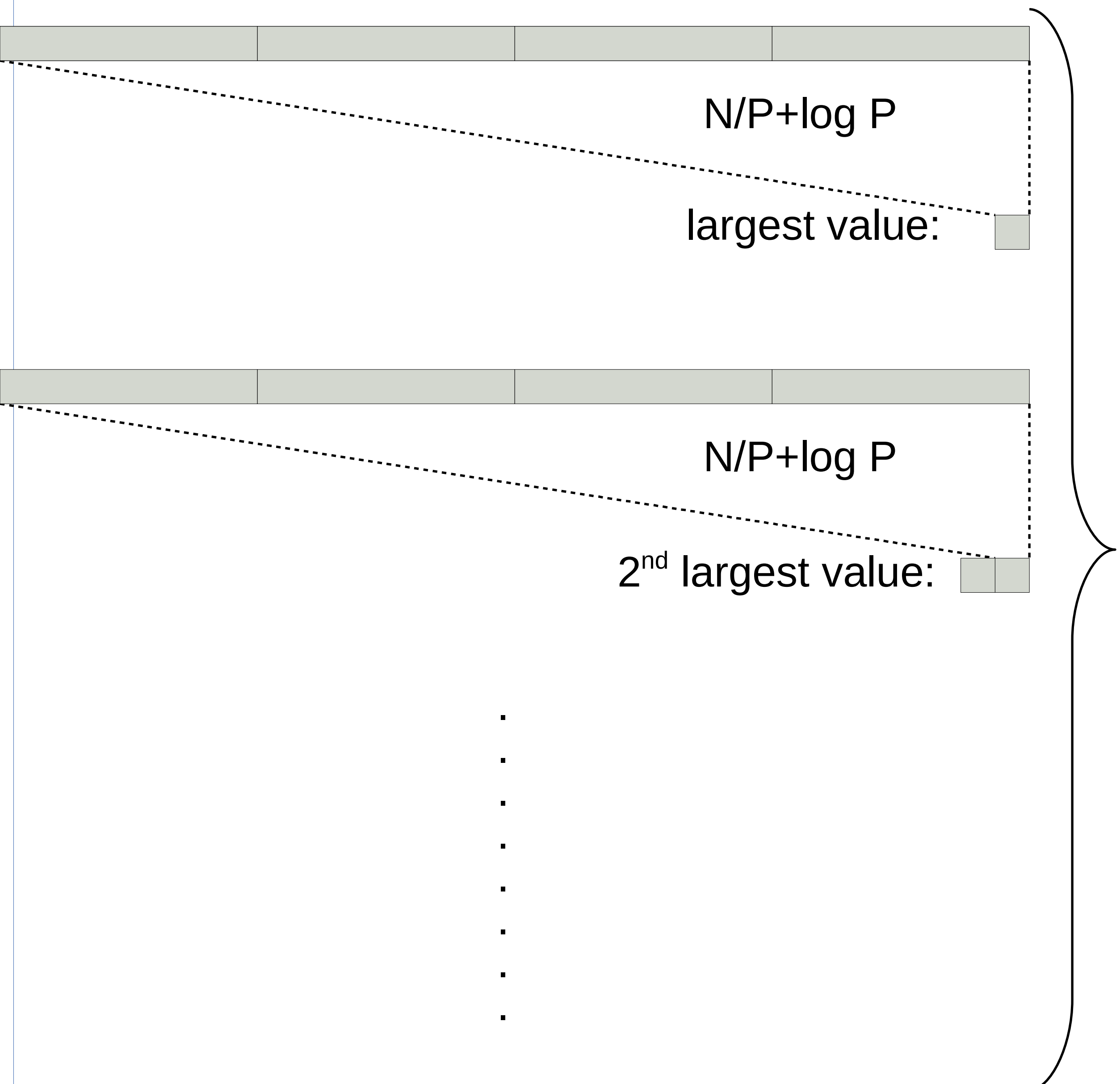
Sequential Sorting: Two Examples

- Quicksort
  - Θ(N log N)
  - Fast in practice
  - Unstable
    - Data with identical keys might end up in a different order
      - Many applications require those data to retain their order
  - Sensitive to median selection
    - Worst case complexity is quadratic
    - Using median of medians is complicated and costly
- Heap sort
  - Θ(N log N)
  - Slower in practice
    - Building and maintaining virtual tree of data: heap
  - Stable
  - Worst case complexity is the same as the average case

Naive Parallel Sort (Don’t Use!)



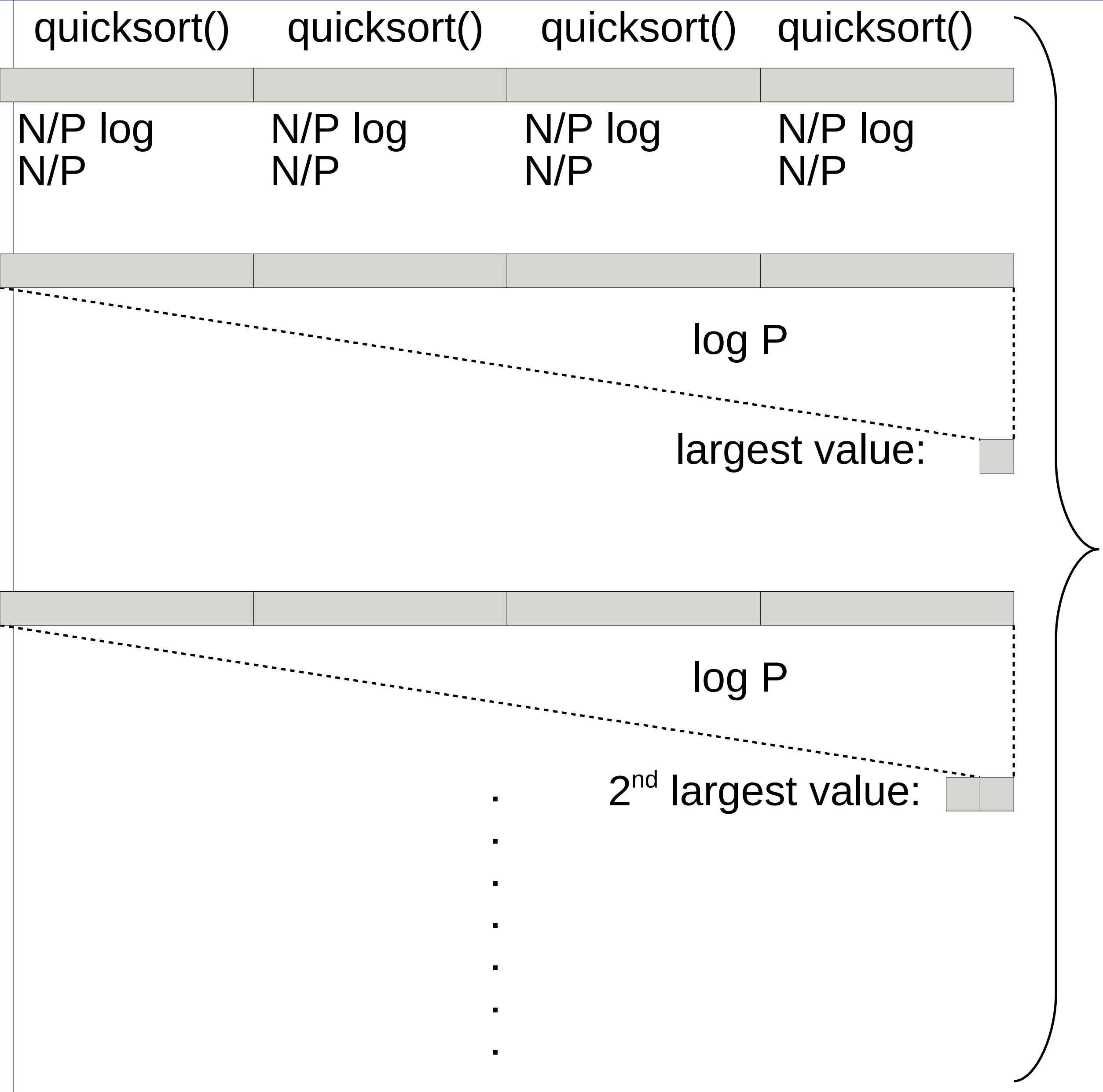
- Partitioning is simple:
  - Each process “p” gets N/P elements

Repeat for each of N elements

- Complexity
  - $(N/P+\log P) * N = N^2/P + N \log P$
- Very simple implementation:

```
for (e = 0; e < N; ++e)
  MPI_Reduce(…, MPI_MAX)
```

Improved Naive Parallel Sort

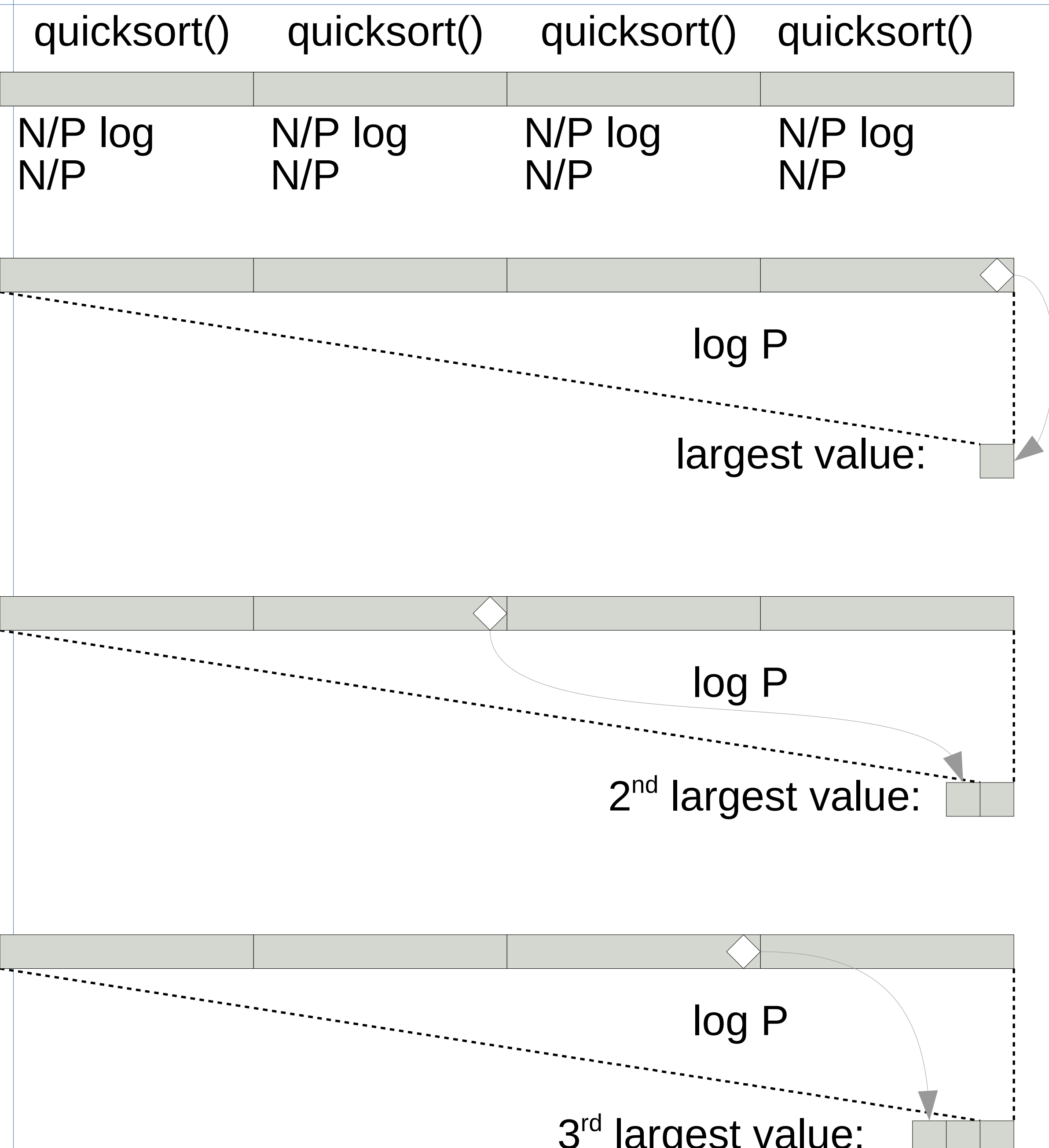


Repeat for each of N elements

- Complexity
  - N/P log N/P + (log P) * N =
    = N/P log N/P + N log P
- Very simple implementation:

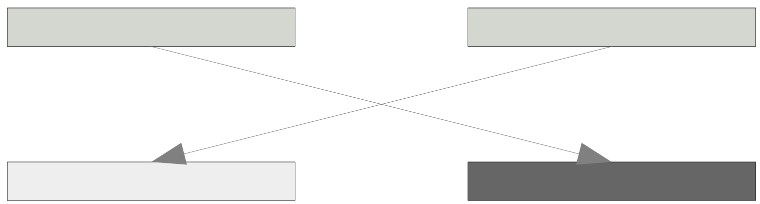
```
quicksort();
for (e = 0; e < N; ++e)
  MPI_Reduce(…, MPI_MAX)
```

Main Problem with Naive Implementations



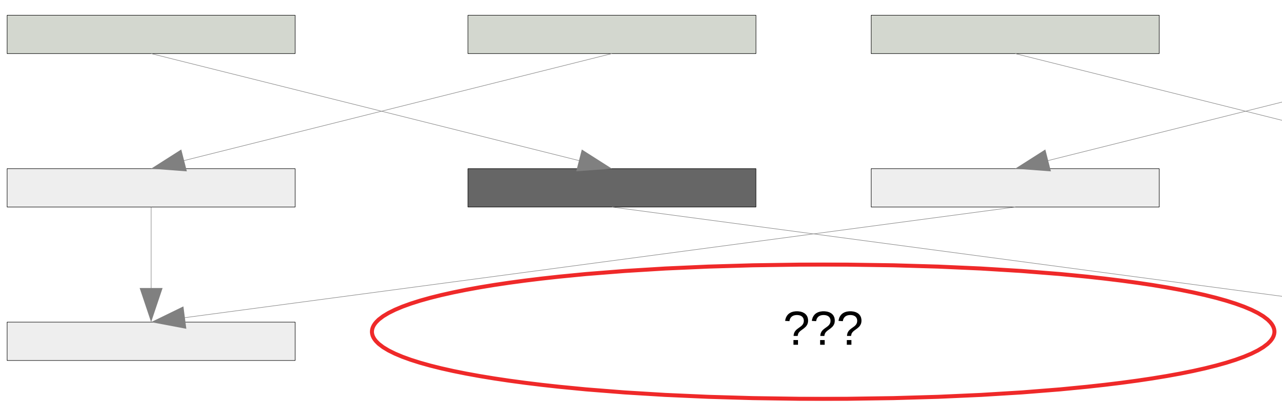
- We must keep track of location of the largest element: `MPI_Reduce(…, MPI_MAXLOC)`
- We must keep track of number of local elements: `MPI_Reduce(…, local[lastEl])`
- We must keep track of where the value should go: `MPI_Reduce(currentRoot, …)`
- All processes need to know the location: `MPI_Bcast(currentRoot, &maxloc)`

Towards Better Parallel Sort



2 processes

smallest values largest values



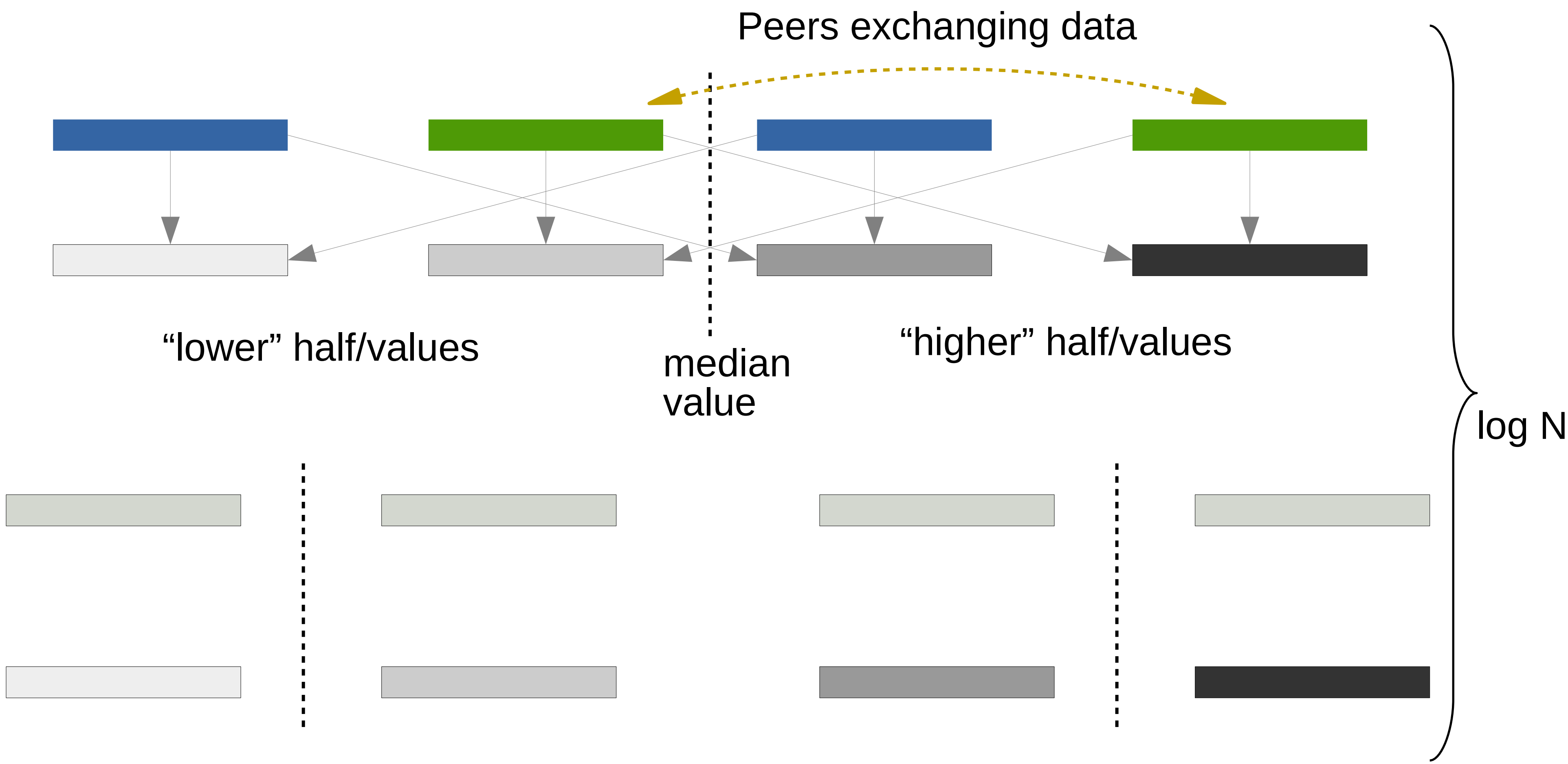
4 processes

???

no invariant guaranteed

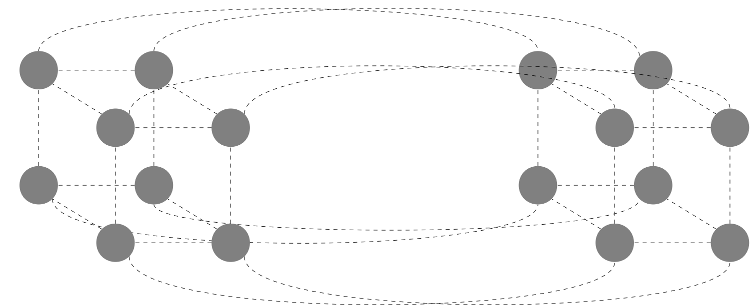
smallest values largest values

Parallel Sort Using a Median: Hyperquicksort



- How to select median?
  1. Pick a process and value at random
  2. Sort values locally and pick a local median
  3. Global communication required for better median
- Keep the local values sorted
  - Initial cost: (N/P log N/P)
  - Merge local old values with global new values: (N/P)

Divisibility, Network, and Median Selection



- Ideally
  - N is power of 2
    - Good load balancing
  - P is power of 2
    - Easy to find partner processor at each recursion level
  - Network is a hypercube
    - Easy to translate logical processor numbers to physical addresses
    - Bandwidth of the network grows with the network size
    - Latency to send a message increases slowly with network size
- Median selection
  - Local median is easy to find
    - Local values are kept sorted
  - Local median is usually not a global one
    - Imagine data that is already sorted
  - Bad median will create a load imbalance
    - Local data is no longer power of 2
    - It is costly to rebalance the load after every median