Distributed Dense Linear Algebra on Heterogeneous Architectures

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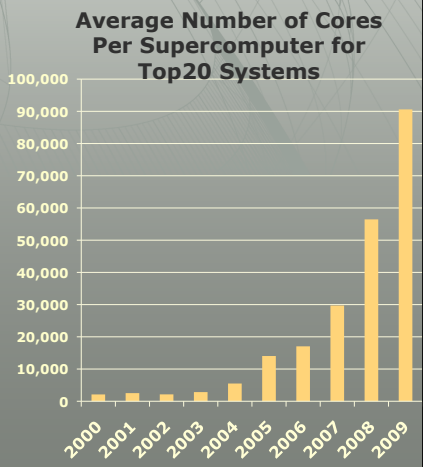
Factors that Necessitate to Redesign of Our Software

- **Steepness of the ascent from terascale to petascale to exascale**
- Extreme parallelism and hybrid design
  - Preparing for million/billion way parallelism
- Tightening memory/bandwidth bottleneck
  - Limits on power/clock speed implication on multicore
  - Reducing communication will become much more intense
  - Memory per core changes, byte-to-flop ratio will change
- Necessary Fault Tolerance
  - MTTF will drop
  - Checkpoint/restart has limitations

Software infrastructure does not exist today



Linpack Evolution

Software/Algorithms follow hardware evolution in time		
LINPACK (70's) (Vector operations)		Rely on - Level-1 BLAS operations
LAPACK (80's) (Blocking, cache friendly)		Rely on - Level-3 BLAS operations
ScaLAPACK (90's) (Distributed Memory)		Rely on - PBLAS Mess Passing
PLASMA (00's) New Algorithms (many-core friendly)		Rely on - a DAG/scheduler - block data layout - some extra kernels

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Linpack Evolution

Software/Algorithms follow hardware evolution in time		
PLASMA (00's) New Algorithms (many-core friendly)		Rely on - a DAG/scheduler - block data layout - some extra kernels

Those new algorithms
- have a very **low granularity**, they scale very well (multicore, petascale computing, …)
- **removes of dependencies** among the tasks, (multicore, distributed computing)
- **avoid latency** (distributed computing, out-of-core)
- **rely on fast kernels**

Those new algorithms need new kernels and rely on efficient scheduling algorithms.

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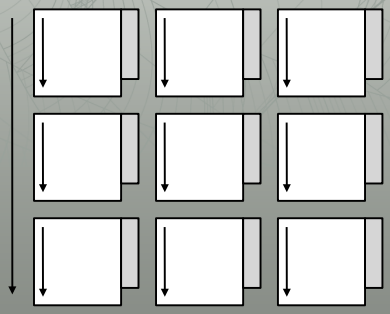
The algorithmic challenge



- Tiled
- Column-major / column-major
- Flat (no indirection)

- Asynchronicity
  - Avoid fork-join (Bulk sync design)
- Dynamic Scheduling
  - Out of order execution
- Fine Granularity
  - Independent block operations
- Locality of Reference
  - Data storage – Block Data Layout

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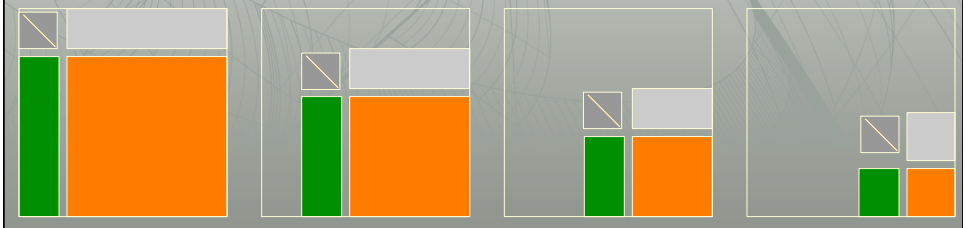
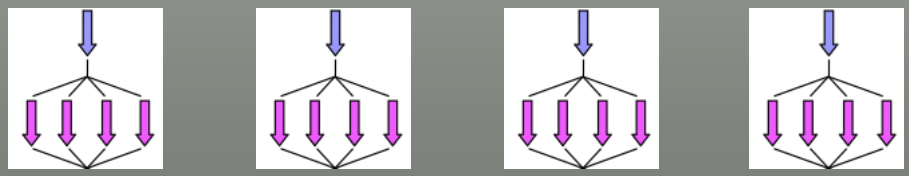
LAPACK LU





- Fork-join, bulk synchronous processing

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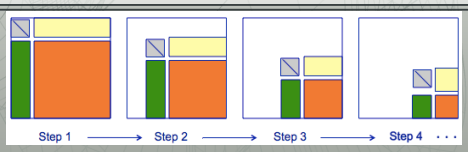
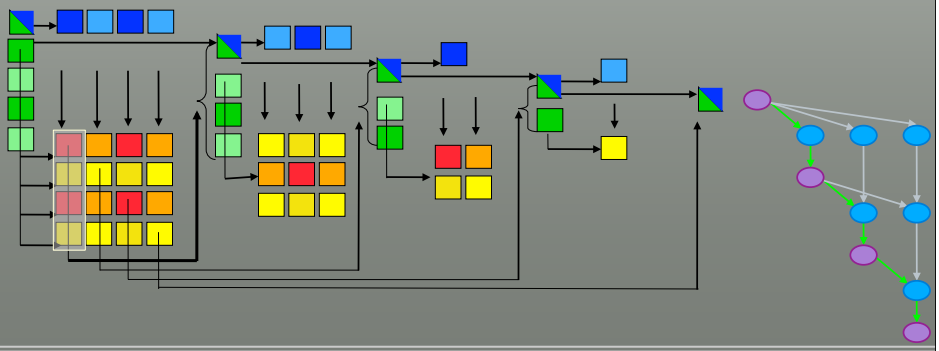
Parallel Tasks in LU



- Expose the intrinsic parallelism: break into smaller tasks and remove dependencies



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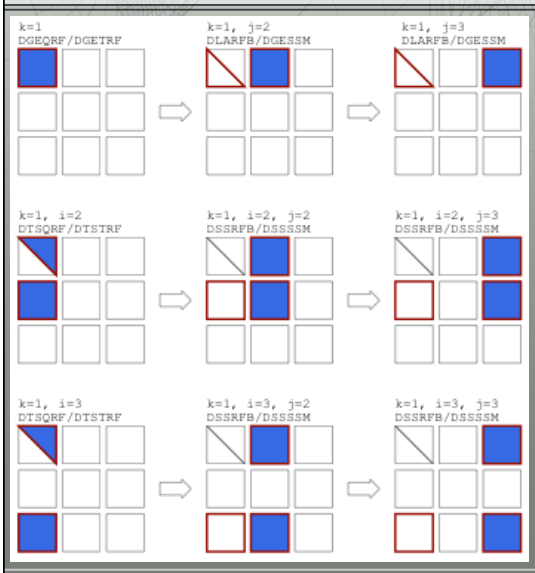
Expose the intrinsic parallelism



- The three amigos
- Example of QR factorization
  - 4 basic kernels
- LU identical to QR except using different kernels
- Cholesky slightly different due to the matrix symmetry

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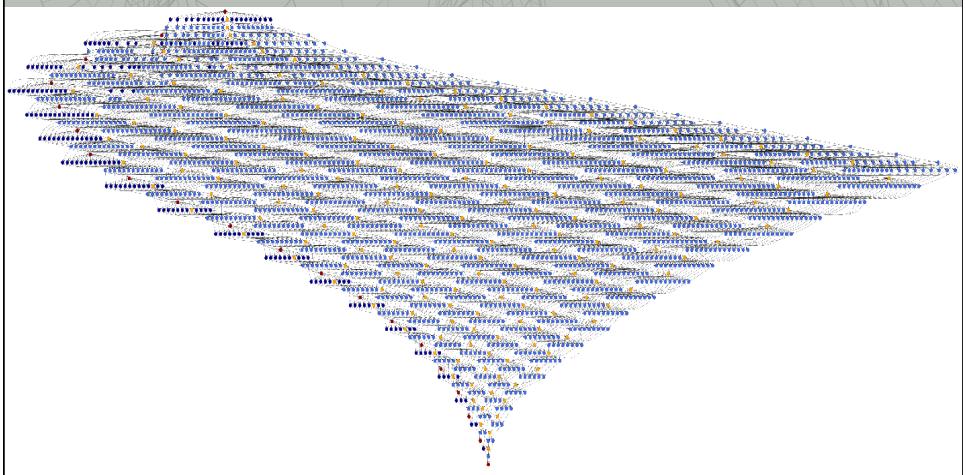
LU DAG representation



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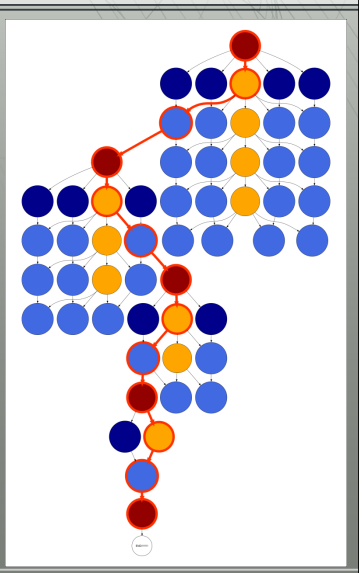
A quite simple problem …

- We would generate the DAG, find the critical path and execute it.
- DAG too large to generate ahead of time
  - Not explicitly generate
  - Dynamically generate the DAG as we go
- Machines will have large number of cores in a distributed fashion
  - Will have to engage in message passing
  - Distributed management
  - Locally have a run time system



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What is PLASMA?

- Dense linear algebra software library
  - Linear systems & least squares (LU, Cholesky, QR/LQ)
  - Eigenvalues & singular values
- Multicore processors
  - Multi-socket multi-core / Shared memory systems
  - NUMA systems
- What is next:
  - GPU acceleration – MAGMA
  - Distributed Memory – DAGuE / DPLASMA

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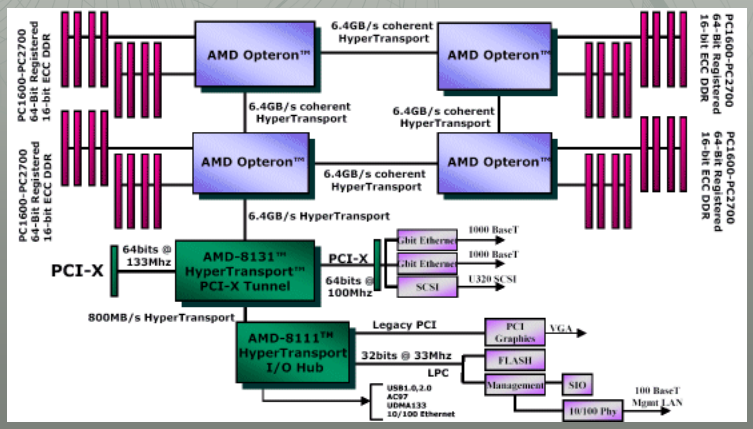
If the current architectures …



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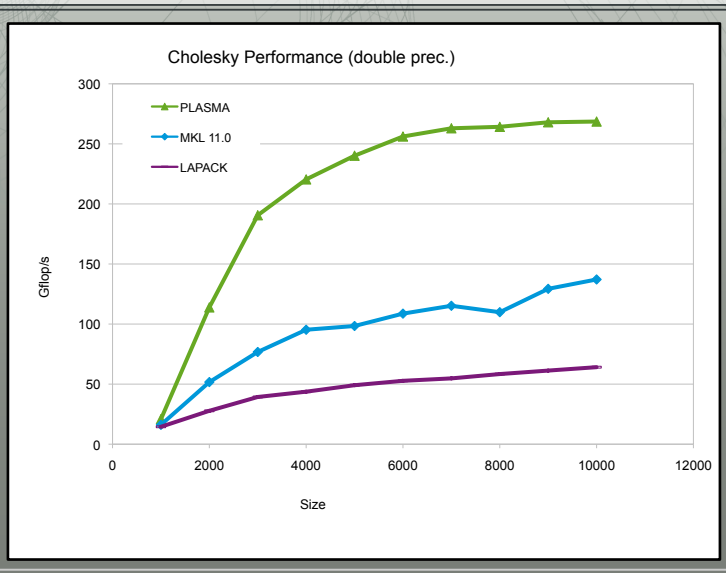
PLASMA Performance (QR, 48 cores)



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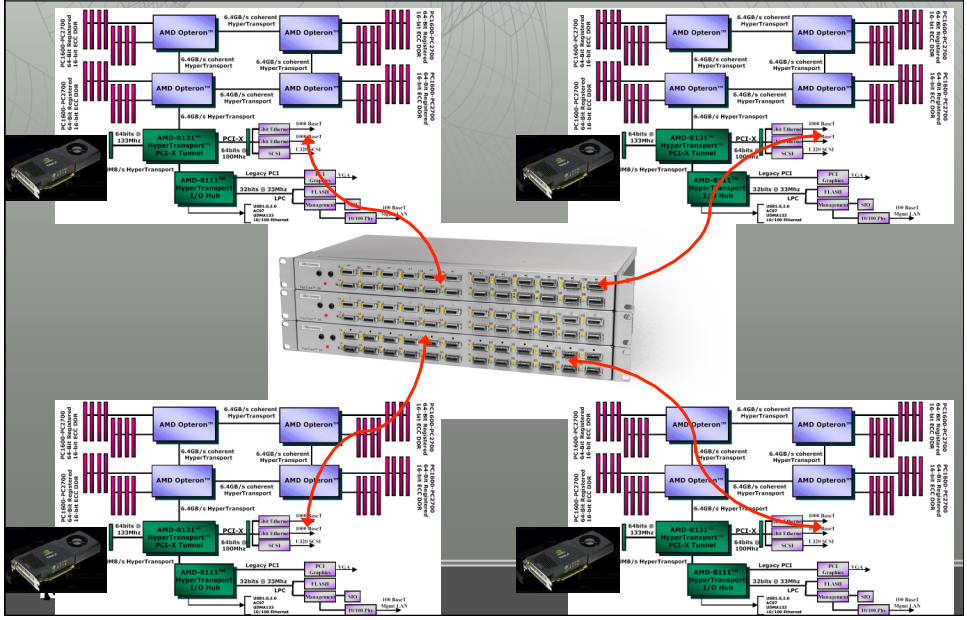
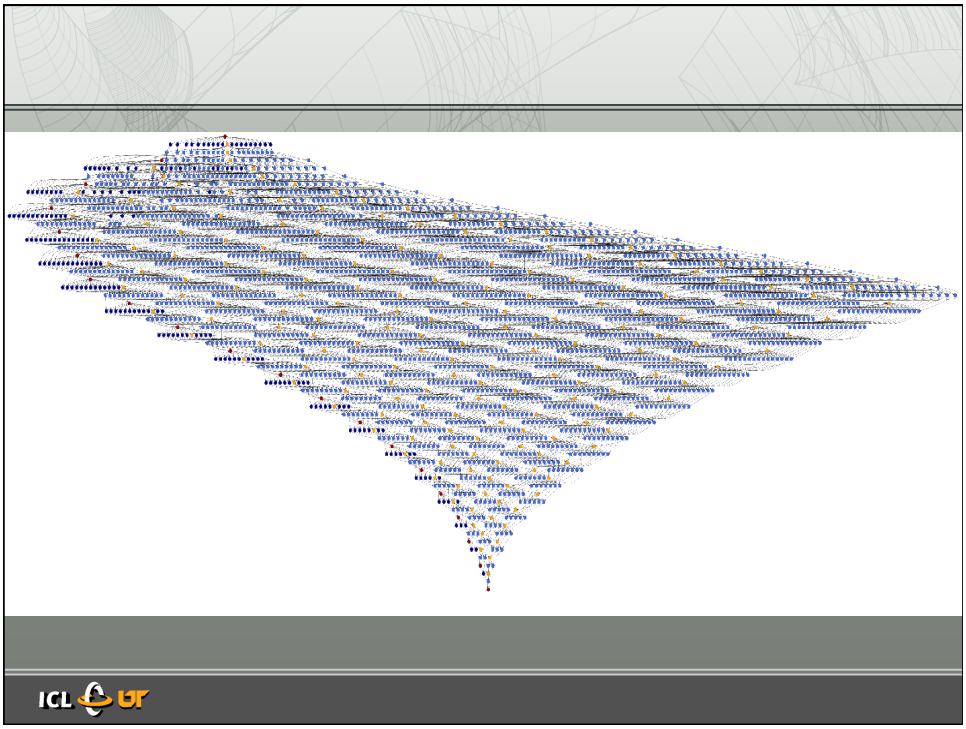
Current Architectures

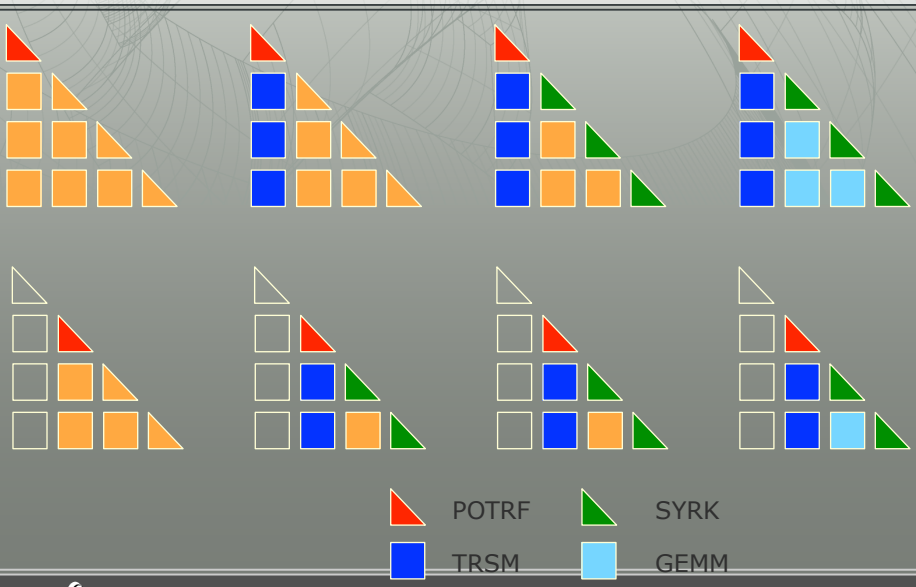




DPLASMA / DAGuE

- DAGuE: the runtime
  - Deploy a DAG on a heterogeneous distributed environment
  - Architecture aware
    - Minimize data movements (in and out the node)
    - Enforce data locality (cache / NUMA / GPU)
  - Move the data across nodes
- DPLASMA: a algebraic description of a DAG
  - Define the data distribution
  - Describe the algorithm at a high level
    - A powerful description language (?)

Cholesky factorization



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Kernels description

```
POTRF(k) (high_priority)
RW T <- type = [tile]
     -> type = [tile]
BODY [core]
  CORE(potrf, (uplo, NB, T, NB,
               &INFO))
END
```

```
SYRK(k,n) (high_priority)
R  A <- type = [tile]
RW T <- type = [tile]
     -> type = [tile]
BODY [core]
  CORE(syrk, (Lower, NoTrans, NB, NB, -1.0,
              A, NB, 1.0, T, NB))
END
```

```
TRSM(k,n) (high_priority)
R  T <- type = [tile]
RW C <- type = [tile]
     -> type = [tile]
BODY [core]
  CORE(trsm, (Right, Lower, Trans, NonUnit,
              NB, NB, 1.0, T, NB, C, NB))
END
```

```
GEMM(k,m,n)
R  A <- type = [tile]
R  B <- type = [tile]
RW C <- type = [tile]
     -> type = [tile]
BODY [core]
  CORE(gemm, (NoTrans, Trans, NB, NB, NB,
              -1.0, B, NB, A, NB, 1.0, C, NB))
END
BODY [cuda]
  ...
END
```

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Algebraic dependencies description

```
POTRF(k)
 k = [0 .. SIZE-1]
: (k / rtileSIZE) % GRIDrows == rowRANK
: (k / ctileSIZE) % GRIDcols == colRANK
 T <- (k == 0) ? A(k, k) : T SYRK(k-1, k)
   -> T TRSM(k, k+1..SIZE-1)
   -> A(k, k)

; (k >= (SIZE - PRI_CHANGE)) ? 10 * (SIZE - k)
* (SIZE - k) * (SIZE - k) : 1000000000
```

```
SYRK(k,n)
 k = [0 .. SIZE-1]
 n = [k+1 .. SIZE-1]
: (n / rtileSIZE) % GRIDrows == rowRANK
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 A <- C TRSM(k, n)
 T <- (k == 0) ? A(n,n) : T SYRK(k-1, n)
   -> (n == k+1) ? T POTRF(k+1) : T SYRK(k+1,n)

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 C <- (k == 0) ? A(n, k) : C GEMM(k-1, n, k)
   -> A SYRK(k, n)
   -> A GEMM(k, n+1..SIZE-1, n)
   -> B GEMM(k, n, k+1..n-1)
   -> A(n, k)

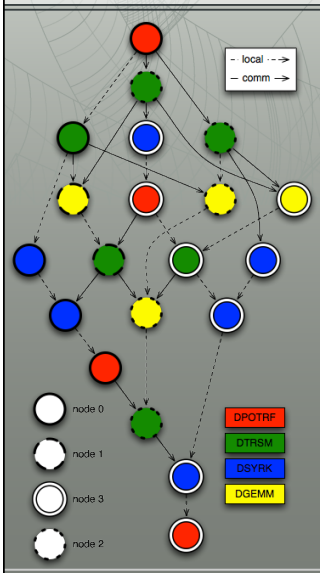
; (n >= (SIZE - PRI_CHANGE)) ? 10 * (SIZE - n)
* (SIZE - n) * (SIZE - n) + 2 : 1000000000
```

```
GEMM(k,m,n) (assoc(k))
 k = [0 .. SIZE]
 m = [k+2 .. SIZE-1]
 n = [k+1 .. m-1]
: (m / rtileSIZE) % GRIDrows == rowRANK
: (n / ctileSIZE) % GRIDcols == colRANK
 A <- C TRSM(k, n)
 B <- C TRSM(k, m)
 C <- (k == 0) ? A(m, n) : C GEMM(k-1, m, n)
   -> (n == k+1) ? C TRSM(k+1, m) :
                   C GEMM(k+1, m, n)

; (m >= (SIZE - PRI_CHANGE)) ? 10 * (SIZE - m)
* (SIZE - m) * (SIZE - m) + 1 : 1000000000
```

ICL UT Execution space | Parallel partitioning | Parameters | Priority

Cholesky (4x4)



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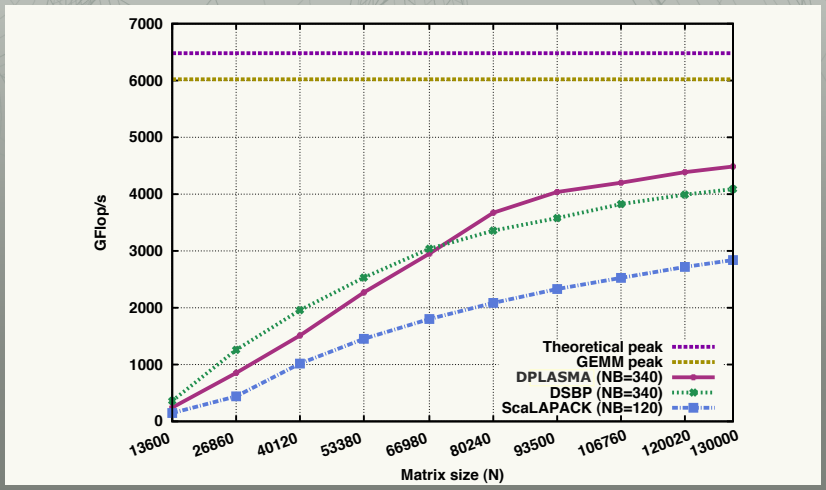
Cholesky (problem size)



Griffon : 81 nodes, 648 cores, Infiniband 20Gbs

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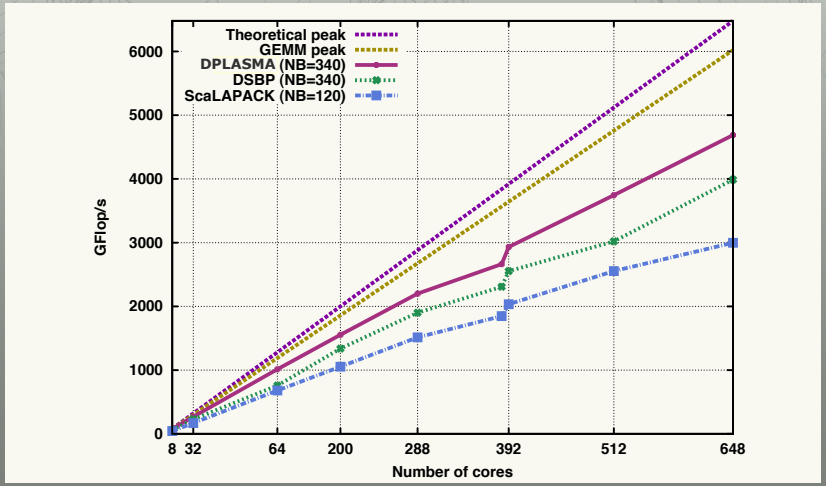
Cholesky (weak scalability)



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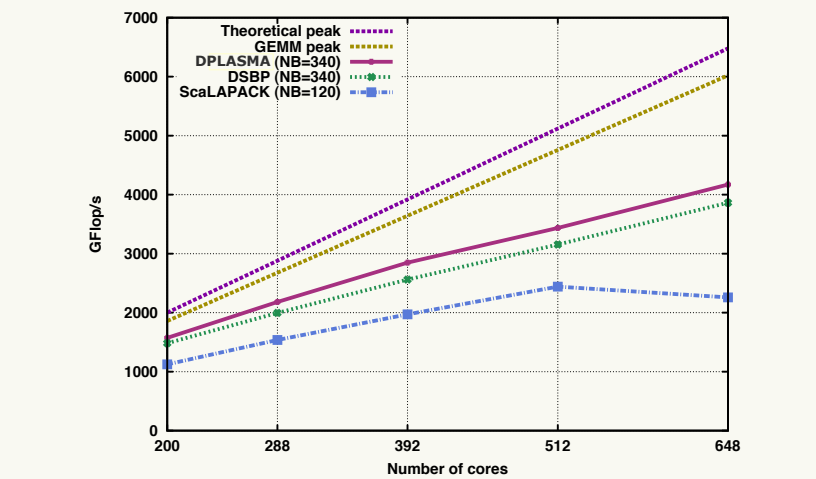
Cholesky (strong scalability)



Computational Cost

```
POTRF(k) (high_priority)
RW T <- type = [tile]
     -> type = [tile]
BODY [core]
  CORE(potrf, (uplo, NB, T, NB,
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END
```

< 2%

```
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```

~3%

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END
```

~3%

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GEMM(k,m,n)
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RW C <- type = [tile]
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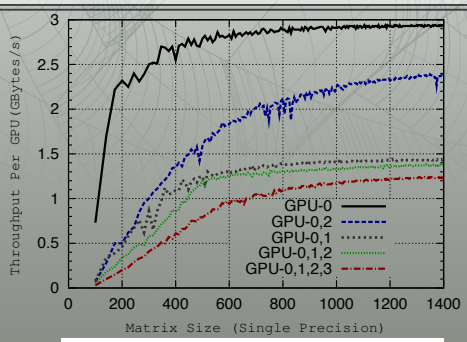
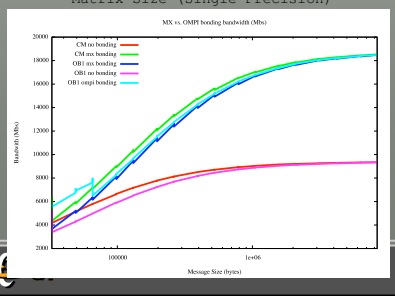
BODY [core]
  CORE(gemm, (NoTrans, Trans, NB, NB, NB,
              -1.0, B, NB, A, NB, 1.0, C, NB))
END
BODY [cuda]
  ...
END
```

> 92%

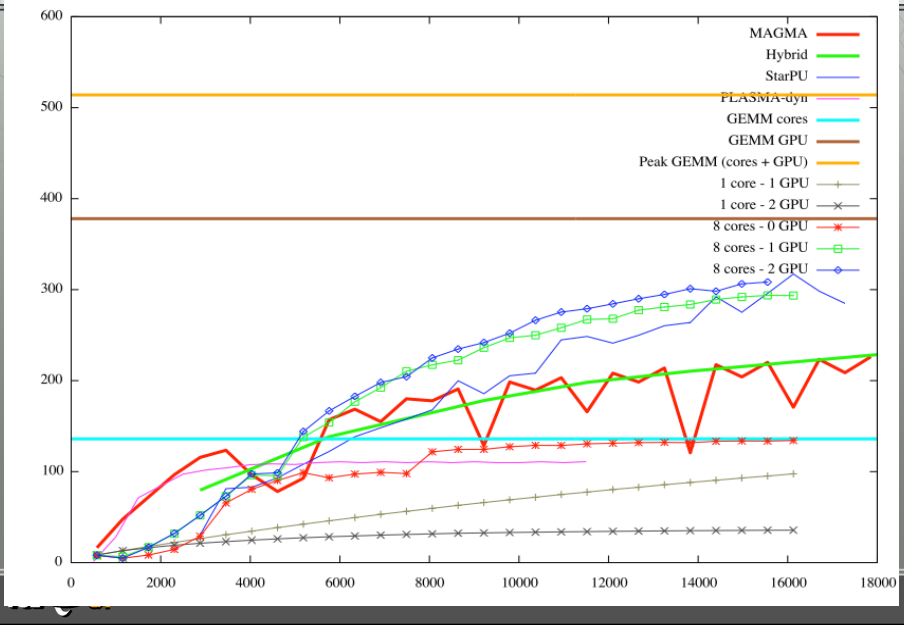
Sharing the resources





- The memory wall
  - Compute intensive kernels
  - Feeding the GPU
  - Moving the data between nodes
- Smart scheduling of the data availability
  - Local vs. Remote

Heterogeneous multi-GPU



Algebraic dependencies description

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 C <- (k == 0) ? A(n, k) : C GEMM(k-1, n, k)
   -> A SYRK(k, n)
   -> A GEMM(k, n+1..SIZE-1, n)
   -> B GEMM(k, n, k+1..n-1)
   -> A(n, k)

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```

```
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 C <- (k == 0) ? A(m, n) : C GEMM(k-1, m, n)
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                   C GEMM(k+1, m, n)

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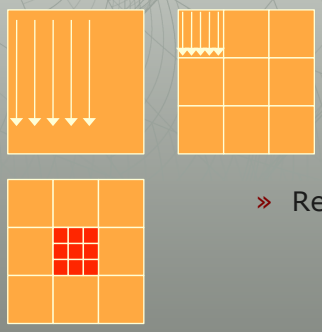
```

ICL UT Execution space | Parallel partitioning | Parameters | Priority

Features



- Automatic on the fly translation from LAPACK format to the PLASMA internal representation

- Recursive behavior

- Modular approach based on Open MPI components model
  - Easy to change change the scheduler or the data mover

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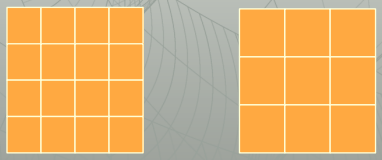
Auto-tuning



- The problem is described based on data dependencies between tiles
  - The definition of the tile is not defined in the algorithm itself
- The runtime is allowed to change the size of the tile based on the current environment
  - Network
  - GPU
  - Cores

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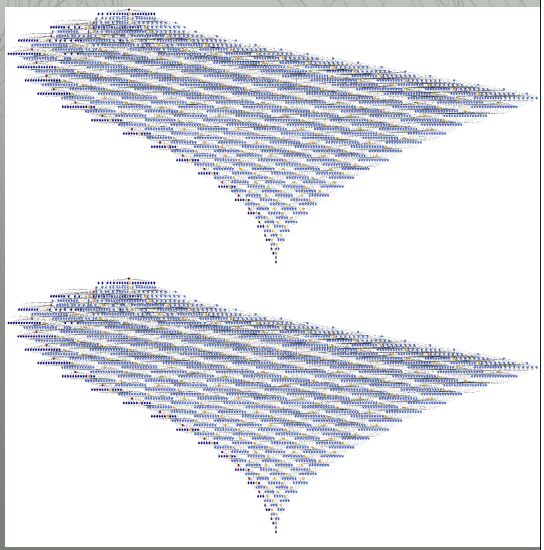
Composability

- Totally remove the need for synchronization
- Follow the data flow between algebraic description of DAGs
- Shorten the time to completion



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Composability