

Algorithms for coping with silent errors

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Outline

- 1 Introduction
- 2 Checkpointing for silent errors
 - Exponential distribution
 - Arbitrary distribution
 - Limited resources
- 3 Checkpointing and verification

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Exascale platforms


- **Hierarchical**
 - 10^5 or 10^6 nodes
 - Each node equipped with 10^4 or 10^3 cores
- **Failure-prone**

MTBF – one node	10 years	120 years
MTBF – platform of 10^6 nodes	5mn	1h

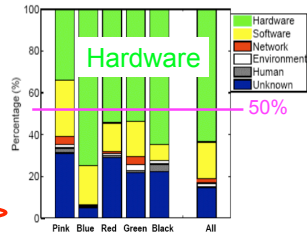
More nodes \Rightarrow Shorter MTBF (Mean Time Between Failures)

Error sources (courtesy Franck Cappello)

Sources of failures

- Analysis of error and failure logs
- In 2005 (Ph. D. of CHARNG-DA LU) : “**Software** halts account for the most number of outages (59-84 percent), and take the shortest time to repair (0.6-1.5 hours). Hardware problems, albeit rarer, need 6.3-100.7 hours to solve.”
- In 2007 (Garth Gibson, ICPP Keynote): 
- In 2008 (Oliner and J. Stearley, DSN Conf.):

Type	Raw		Filtered	
	Count	%	Count	%
Hardware	174,586,516	98.04	1,999	18.78
Software	144,899	0.08	6,814	64.01
Indeterminate	3,350,044	1.88	1,832	17.21



Relative frequency of root cause by system type.

Software errors: Applications, OS bug (kernel panic), communication libs, File system error and other.

Hardware errors, Disks, processors, memory, network

Conclusion: Both Hardware and Software failures have to be considered

Definitions

- Instantaneous error detection \Rightarrow fail-stop failures, e.g. resource crash
- Silent errors (data corruption) \Rightarrow detection latency

Silent error detected only when the corrupt data is activated

- Includes some software faults, some hardware errors (soft errors in L1 cache), double bit flip
- Cannot always be corrected by ECC memory

Quotes

- Soft Error: An unintended change in the state of an electronic device that alters the information that it stores without destroying its functionality, e.g. a bit flip caused by a cosmic-ray-induced neutron. (Hengartner et al., 2008)
- SDC occurs when incorrect data is delivered by a computing system to the user without any error being logged (Cristian Constantinescu, AMD)
- **Silent errors are the black swan of errors** (Marc Snir)

Should we be afraid? (courtesy Al Geist)

Fear of the Unknown

Hard errors – permanent component failure either HW or SW
(hung or crash)

Transient errors – a blip or short term failure of either HW or SW

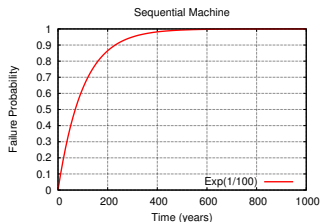
Silent errors – undetected errors either hard or soft, due to lack of detectors for a component or inability to detect (transient effect too short). Real danger is that answer may be incorrect but the user wouldn't know.

**Statistically, silent error rates are increasing.
Are they really? Its fear of the unknown**

Are silent errors really a problem
or just monsters under our bed?



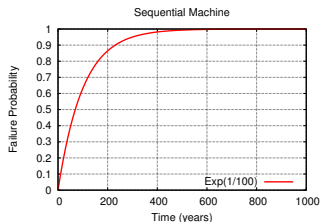
Failure distributions: (1) Exponential



Exp(λ): Exponential distribution law of parameter λ :

- Pdf: $f(t) = \lambda e^{-\lambda t} dt$ for $t \geq 0$
- Cdf: $F(t) = 1 - e^{-\lambda t}$
- Mean = $\frac{1}{\lambda}$

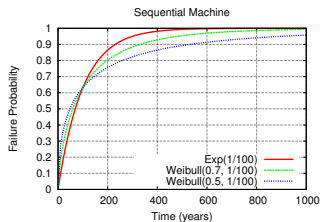
Failure distributions: (1) Exponential



X random variable for $Exp(\lambda)$ failure inter-arrival times:

- $\mathbb{P}(X \leq t) = 1 - e^{-\lambda t}$ (by definition)
- **Memoryless property:** $\mathbb{P}(X \geq t + s | X \geq s) = \mathbb{P}(X \geq t)$
at any instant, time to next failure does not depend upon time elapsed since last failure
- Mean Time Between Failures (MTBF) $\mu = \mathbb{E}(X) = \frac{1}{\lambda}$

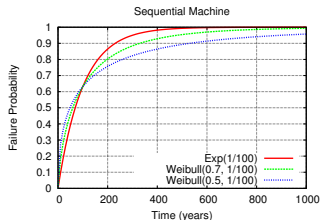
Failure distributions: (2) Weibull



Weibull(k, λ): Weibull distribution law of shape parameter k and scale parameter λ :

- Pdf: $f(t) = k\lambda(t\lambda)^{k-1}e^{-(\lambda t)^k} dt$ for $t \geq 0$
- Cdf: $F(t) = 1 - e^{-(\lambda t)^k}$
- Mean = $\frac{1}{\lambda}\Gamma(1 + \frac{1}{k})$

Failure distributions: (2) Weibull



X random variable for $Weibull(k, \lambda)$ failure inter-arrival times:

- If $k < 1$: failure rate decreases with time
 "infant mortality": defective items fail early
- If $k = 1$: $Weibull(1, \lambda) = Exp(\lambda)$ constant failure time

Failure distributions: with several processors

- Processor (or node): any entity subject to failures
⇒ approach **agnostic to granularity**
- If the MTBF is μ_{ind} with one processor,
what is its value μ_p with p processors?
- Well, it depends ☹️

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- Well, it depends 😞

With rejuvenation

- Rebooting all p processors after a failure
- Platform failure distribution
⇒ minimum of p IID processor distributions
- With p distributions $Exp(\lambda)$:

$$\min_{1..p} (Exp(\lambda)) = Exp(p\lambda)$$

- With p distributions $Weibull(k, \lambda)$:

$$\min_{1..p} (Weibull(k, \lambda)) = Weibull(k, p^{1/k} \lambda)$$

Without rejuvenation (= real life)

- Rebooting only faulty processor
- Platform failure distribution
⇒ superposition of p IID processor distributions

Theorem: $\mu_p = \frac{\mu_{\text{ind}}}{p}$ for arbitrary distributions

Lesson learnt for fail-stop failures

(Not so) Secret data

- Tsubame 2: 962 failures during last 18 months so $\mu = 13$ hrs
- Blue Waters: 2-3 node failures per day
- Titan: a few failures per day
- Tianhe 2: wouldn't say

$$T_{\text{opt}} = \sqrt{2\mu C} \quad \Rightarrow \quad \text{WASTE}_{\text{opt}} \approx \sqrt{\frac{2C}{\mu}}$$

Petascale:	$C = 20$ min	$\mu = 24$ hrs	$\Rightarrow \text{WASTE}_{\text{opt}} = 17\%$
Scale by 10:	$C = 20$ min	$\mu = 2.4$ hrs	$\Rightarrow \text{WASTE}_{\text{opt}} = 53\%$
Scale by 100:	$C = 20$ min	$\mu = 0.24$ hrs	$\Rightarrow \text{WASTE}_{\text{opt}} = 100\%$

Lesson learnt for fail-stop failures

(Also) Secret data

- Tsubame: 962 failures during last 18 months so far 13 hrs
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Exascale \neq Petascale $\times 1000$

Need more reliable components

Need to checkpoint faster

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Silent errors:
detection latency \Rightarrow additional problems

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Application-specific methods

- ABFT: dense matrices / fail-stop, extended to sparse / silent. Limited to one error detection and/or correction in practice
- Asynchronous (chaotic) iterative methods (old work)
- Partial differential equations: use lower-order scheme as verification mechanism (detection only, Benson, Schmit and Schreiber)
- FT-GMRES: inner-outer iterations (Hoemmen and Heroux)
- PCG: orthogonalization check every k iterations, re-orthogonalization if problem detected (Sao and Vuduc)
- ... Many others

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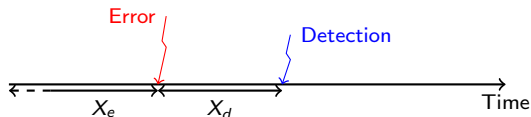
Checkpointing for silent errors

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Checkpointing and verification

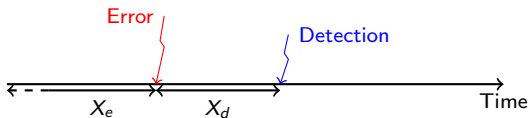
General-purpose approach



Error and detection latency

- Last checkpoint may have saved an already corrupted state
- Saving k checkpoints (Lu, Zheng and Chien):
 - ① Which checkpoint to roll back to?
 - ② Critical failure when all live checkpoints are invalid

Optimal period?



Error and detection latency

- X_e inter arrival time between errors; mean time μ_e
- X_d error detection time; mean time μ_d
- Assume X_d and X_e independent

Notations

- C checkpointing time
- R recovery time
- W total work
- w some piece of work

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For one chunk

When X_e follows an Exponential law of parameter $\lambda_e = \frac{1}{\mu_e}$, in order to execute a total work of $w + C$, we need:

- Probability of execution without error

$$\begin{aligned} \mathbb{E}(T(w)) &= e^{-\lambda_e(w+C)} (w + C) \\ &+ (1 - e^{-\lambda_e(w+C)}) (\mathbb{E}(T_{lost}) + \mathbb{E}(X_d) + \mathbb{E}(T_{rec}) + \mathbb{E}(T(w))) \end{aligned}$$

- Probability of error during $w + C$
- Execution time with an error

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Focus on time lost due to an error:

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This is the time elapsed between the completion of last checkpoint and the error

$$\begin{aligned}\mathbb{E}(T_{lost}) &= \int_0^{\infty} x \mathbb{P}(X = x | X < w + C) dx \\ &= \frac{1}{\mathbb{P}(X < w + C)} \int_0^{w+C} x \lambda_e e^{-\lambda_e x} dx \\ &= \frac{1}{\lambda_e} - \frac{w + C}{e^{\lambda_e(w+C)} - 1}\end{aligned}$$

Focus on time lost due to an error:

$$\mathbb{E}(T_{lost}) + \mathbb{E}(X_d) + \mathbb{E}(T_{rec})$$

This is the time needed for error detection, $\mathbb{E}(X_d) = \mu_d$

Focus on time lost due to an error:

$$\mathbb{E}(T_{lost}) + \mathbb{E}(X_d) + \mathbb{E}(T_{rec})$$

This is the time to recover from the error (there can be a fault during recovery):

$$\begin{aligned}\mathbb{E}(T_{rec}) &= e^{-\lambda_e R} R \\ &+ (1 - e^{-\lambda_e R})(\mathbb{E}(R_{lost}) + \mathbb{E}(X_d) + \mathbb{E}(T_{rec}))\end{aligned}$$

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So finally, $\mathbb{E}(T_{rec}) = (e^{\lambda_e R} - 1)(\mu_e + \mu_d)$

At the end of the day,

$$\mathbb{E}(T(w)) = e^{\lambda_e R} (\mu_e + \mu_d) (e^{\lambda_e(w+C)} - 1)$$

This is the exact solution!

For multiple chunks

Using n chunks of size w_i (with $\sum_{i=1}^n w_i = W$), we have:

$$\mathbb{E}(T(W)) = K \sum_{i=1}^n (e^{\lambda_e(w_i+C)} - 1)$$

with K constant.

Independent of μ_d !

Minimum when all the w_i 's are equal to $w = W/n$.

Optimal n can be found by differentiation

A good approximation is $w = \sqrt{2\mu_e C}$ (Young's formula)

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○ Exponential distribution

● Arbitrary distribution

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Arbitrary distributions

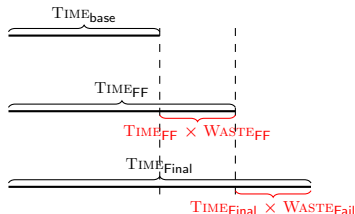
Extend results when X_e follows an arbitrary distribution of mean μ_e

Framework

Waste: fraction of time not spent for useful computations

Waste

- $\text{TIME}_{\text{base}}$: application base time
- TIME_{FF} : with periodic checkpoints but failure-free
- $\text{TIME}_{\text{Final}}$: expectation of time with failures



$$(1 - \text{WASTE}_{\text{FF}})\text{TIME}_{\text{FF}} = \text{TIME}_{\text{base}}$$

$$(1 - \text{WASTE}_{\text{Fail}})\text{TIME}_{\text{Final}} = \text{TIME}_{\text{FF}}$$

$$\text{WASTE} = \frac{\text{TIME}_{\text{Final}} - \text{TIME}_{\text{base}}}{\text{TIME}_{\text{Final}}}$$

$$\text{WASTE} = 1 - (1 - \text{WASTE}_{\text{FF}})(1 - \text{WASTE}_{\text{Fail}})$$

Back to our model

We can show that

$$\text{WASTE}_{\text{FF}} = \frac{C}{T}$$

$$\text{WASTE}_{\text{Fail}} = \frac{\frac{T}{2} + R + \mu_d}{\mu_e}$$

Only valid if $\frac{T}{2} + R + \mu_d \ll \mu_e$.

Then the waste is minimized for

$$T_{\text{opt}} = \sqrt{2(\mu_e - (R + \mu_d))C} \approx \sqrt{2\mu_e C}$$

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Summary

Theorem

- *Best period is $T_{opt} \approx \sqrt{2\mu_e C}$*
- *Independent of X_d*

Limitation of this model

Analytical optimal solutions, valid for arbitrary distributions, without any knowledge on X_d except its mean

However, if X_d can be arbitrary large:

- Do not know how far to roll back in time
- Need to store all checkpoints taken during execution

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The case with limited resources

Assume that we can only save **the last k checkpoints**

Definition (Critical failure)

Error detected when all checkpoints contain corrupted data.
Happens with probability \mathbb{P}_{risk} during whole execution.

The case with limited resources

\mathbb{P}_{risk} decreases when T increases (when X_d is fixed).

Hence, $\mathbb{P}_{\text{risk}} \leq \varepsilon$ leads to a lower bound T_{\min} on T

We have derived an analytical form for \mathbb{P}_{risk} when X_d follows an Exponential law. We use it as a good(?) approximation for arbitrary laws

Limitation of the model

It is not clear how to detect when the error has occurred
(hence to identify the last valid checkpoint) ☹️ ☹️ ☹️

Need a verification mechanism to check the correctness of the checkpoints. This has an additional cost!

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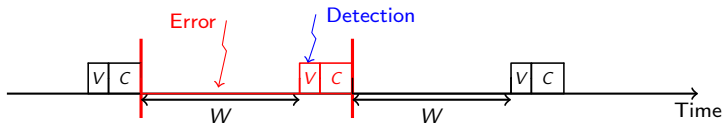
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Checkpointing and verification

Coupling checkpointing and verification

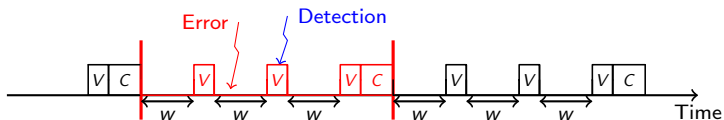
- Verification mechanism of cost V
- Silent errors detected only when verification is executed
- Approach agnostic of the nature of verification mechanism (checksum, error correcting code, coherence tests, etc)
- Fully general-purpose
(application-specific information, if available, can always be used to decrease V)

Base pattern (and revisiting Young/Daly)



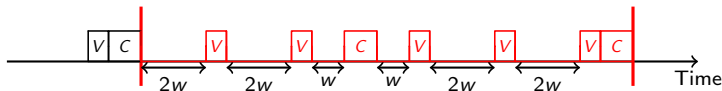
	Fail-stop (classical)	Silent errors
Pattern	$T = W + C$	$S = W + V + C$
$WASTE_{FF}$	$\frac{C}{T}$	$\frac{V+C}{S}$
$WASTE_{fail}$	$\frac{1}{\mu}(D + R + \frac{W}{2})$	$\frac{1}{\mu}(R + W + V)$
Optimal	$T_{opt} = \sqrt{2C\mu}$	$S_{opt} = \sqrt{(C + V)\mu}$
$WASTE_{opt}$	$\sqrt{\frac{2C}{\mu}}$	$2\sqrt{\frac{C+V}{\mu}}$

With $p = 1$ checkpoint and $q = 3$ verifications



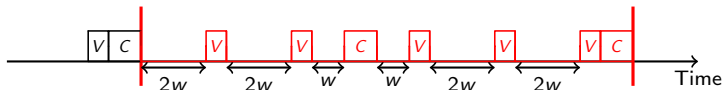
Base Pattern	$p = 1, q = 1$	$WASTE_{opt} = 2\sqrt{\frac{C+V}{\mu}}$
New Pattern	$p = 1, q = 3$	$WASTE_{opt} = 2\sqrt{\frac{4(C+3V)}{6\mu}}$

BALANCED ALGORITHM



- p checkpoints and q verifications, $p \leq q$
- $p = 2$, $q = 5$, $S = 2C + 5V + W$
- $W = 10w$, six chunks of size w or $2w$
- May store invalid checkpoint (error during third chunk)
- After successful verification in fourth chunk, preceding checkpoint is valid
- Keep only two checkpoints in memory and avoid any fatal failure

BALANCED ALGORITHM



- ① (proba $2w/W$) $T_{\text{lost}} = R + 2w + V$
- ② (proba $2w/W$) $T_{\text{lost}} = R + 4w + 2V$
- ③ (proba w/W) $T_{\text{lost}} = 2R + 6w + C + 4V$
- ④ (proba w/W) $T_{\text{lost}} = R + w + 2V$
- ⑤ (proba $2w/W$) $T_{\text{lost}} = R + 3w + 2V$
- ⑥ (proba $2w/W$) $T_{\text{lost}} = R + 5w + 3V$

$$\text{WASTE}_{\text{opt}} \approx 2\sqrt{\frac{7(2C + 5V)}{20\mu}}$$

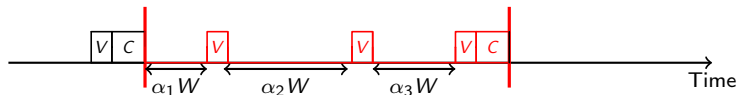
Analysis

- $S = pC + qV + pqw \ll \mu$
- $\text{WASTE}_{\text{FF}} = \frac{o_{\text{ff}}}{S}$, where $o_{\text{ff}} = pC + qV$
- $\text{WASTE}_{\text{Fail}} = \frac{T_{\text{lost}}}{\mu}$, where $T_{\text{lost}} = f_{\text{re}}S + \beta$
 - f_{re} : *fraction of work that is re-executed*
 - β : constant, linear combination of C , V and R
 - $f_{\text{re}} = \frac{7}{20}$ when $p = 2, q = 5$

$$S_{\text{opt}} = \sqrt{\frac{o_{\text{ff}}}{f_{\text{re}}}} \times \sqrt{\mu} + o(\sqrt{\mu})$$

$$\text{WASTE}_{\text{opt}} = 2\sqrt{o_{\text{ff}}f_{\text{re}}} \sqrt{\frac{1}{\mu}} + o\left(\sqrt{\frac{1}{\mu}}\right)$$

Computing f_{re} when $p = 1$

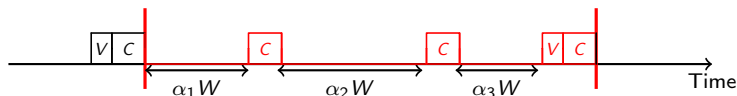


Theorem

The minimal value of $f_{re}(1, q)$ is obtained for same-size chunks

- $f_{re}(1, q) = \sum_{i=1}^q \left(\alpha_i \sum_{j=1}^i \alpha_j \right)$
- Minimal when $\alpha_i = 1/q$
- In that case, $f_{re}(1, q) = \frac{q+1}{2q}$

Computing f_{re} when $p \geq 1$



Theorem

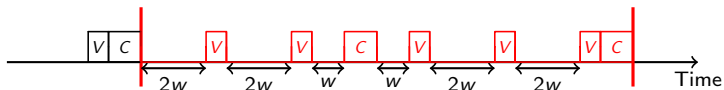
$f_{re}(p, q) \geq \frac{p+q}{2pq}$, bound is matched by BALANCEDALGORITHM.

- Assess gain due to the $p - 1$ intermediate checkpoints
- $f_{re}^{(1)} - f_{re}^{(p)} = \sum_{i=1}^p \left(\alpha_i \sum_{j=1}^{i-1} \alpha_j \right)$
- Maximal when $\alpha_i = 1/p$ for all i
- In that case, $f_{re}^{(1)} - f_{re}^{(p)} = (p - 1)/p^2$
- Now best with equipartition of verifications too
- In that case, $f_{re}^{(1)} = \frac{q+1}{2q}$ and $f_{re}^{(p)} = \frac{q+1}{2q} - \frac{p-1}{2p} = \frac{q+p}{2pq}$

Choosing optimal pattern

- Let $V = \gamma C$, where $0 < \gamma \leq 1$
- $o_{\text{ff}}f_{\text{re}} = \frac{p+q}{2pq}(pC + qV) = C \times \frac{p+q}{2} \left(\frac{1}{q} + \frac{\gamma}{p} \right)$
- Given γ , minimize $\frac{p+q}{2} \left(\frac{1}{q} + \frac{\gamma}{p} \right)$ with $1 \leq p \leq q$, and p, q taking integer values
- Let $p = \lambda \times q$. Then $\lambda_{\text{opt}} = \sqrt{\gamma} = \sqrt{\frac{V}{C}}$

Summary



- BALANCEDALGORITHM optimal when $C, R, V \ll \mu$
- Keep only 2 checkpoints in memory/storage
- Closed-form formula for $WASTE_{opt}$
- Given C and V , choose optimal pattern
- Gain of up to 20% over base pattern

Conclusion

- Soft errors difficult to cope with, even for divisible workloads
- Investigate graphs of computational tasks
- Combine checkpointing and application-specific techniques (ABFT)
- Multi-criteria soptimization problem
execution time/energy/reliability
best resource usage (performance trade-offs)

Several challenging algorithmic/scheduling problems 😊

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