

# On the Combination of Silent Error Detection and Checkpointing

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ICL Friday Lunch – September 6, 2013



## Introduction

## Optimal Checkpointing strategy

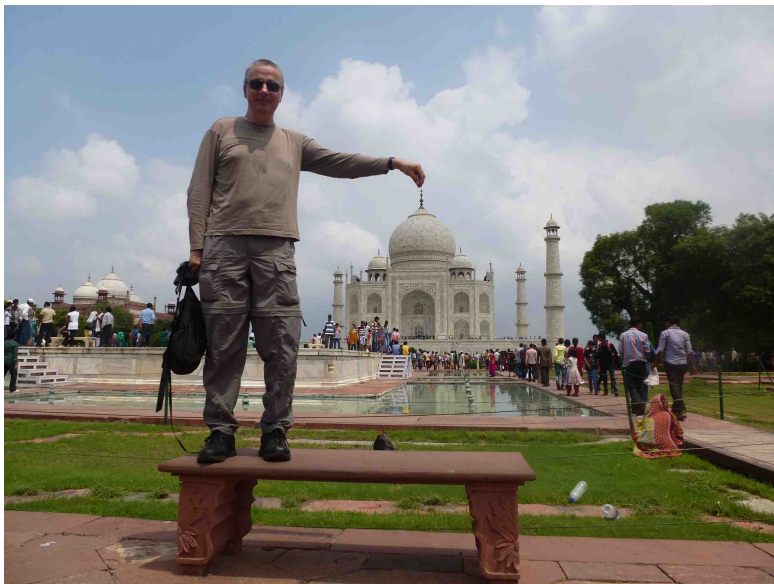
- Exponential  
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distribution

## Limited resources

## Incorporating detection

- $k$  checkpoints  
for 1 verification
- $k$  verifications  
for 1 checkpoint

## Conclusion, future work



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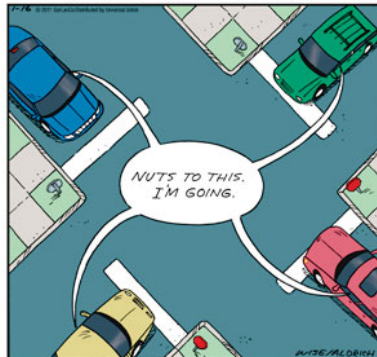
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REAL LIFE ADVENTURES



BY GARY WISE & LANCE ALDRICH



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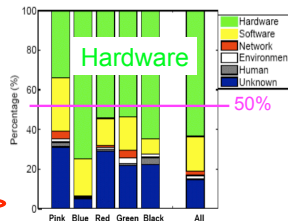
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## Sources of failures

- Analysis of error and failure logs
- In 2005 (Ph. D. of CHARNG-DA LU) : “**Software** halts account for the most number of outages (59-84 percent), and take the shortest time to repair (0.6-1.5 hours). Hardware problems, albeit rarer, need 6.3-100.7 hours to solve.”
- In 2007 (Garth Gibson, ICPP Keynote): 
- In 2008 (Oliner and J. Stearley, DSN Conf.):

Type	Raw		Filtered	
	Count	%	Count	%
Hardware	174 586 516	98.04	1 999	18.78
Software	144,899	0.08	6,814	64.01
Indeterminate	3,350,044	1.88	1,832	17.21



Relative frequency of root  
cause by system type.

Software errors: Applications, OS bug (kernel panic), communication libs, File system error and other.

Hardware errors, Disks, processors, memory, network

Conclusion: Both Hardware and Software failures have to be considered

- Many types of faults: software error, hardware malfunction, memory corruption
- Many possible behaviors: silent, transient, unrecoverable
- Restrict to silent errors
- This includes some software faults, some hardware errors (soft errors in L1 cache), double bit flip
- Silent error detected when corrupt data is activated



- Many types of faults: software error, hardware malfunction, memory corruption
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- Restrict to silent errors
- This includes some software faults, some hardware errors (soft errors in L1 cache), double bit flip
- Silent error detected when corrupt data is activated
- *Silent errors are the black swans of errors* (Marc Snir)

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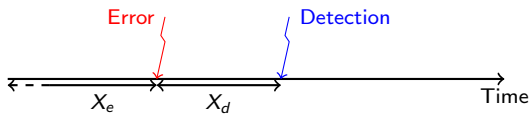


Figure : Error and detection latency.

- $X_e$  inter arrival time between errors; mean time  $\mu_e$
- $X_d$  error detection time; mean time  $\mu_d$
- Assume  $X_d$  and  $X_e$  independent

Y. Robert

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- $C$  checkpointing time
- $R$  recovery time
- $W$  total work
- $w$  some piece of work

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When  $X_e$  follows an Exponential law of parameter  $\lambda_e = \frac{1}{\mu_e}$ , in order to execute a total work of  $w + C$ , we need:

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- Probability of execution without error

$$\mathbb{E}(T(w)) = e^{-\lambda_e(w+C)}(w+C) + (1 - e^{-\lambda_e(w+C)}) (\mathbb{E}(T_{lost}) + \mathbb{E}(X_d) + \mathbb{E}(T_{rec}) + \mathbb{E}(T(w)))$$

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- Probability of error during  $w + C$

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- Probability of error during  $w + C$
- Execution time with an error



$$\mathbb{E}(T_{lost}) + \mathbb{E}(X_d) + \mathbb{E}(T_{rec})$$

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$$\mathbb{E}(T_{lost}) + \mathbb{E}(X_d) + \mathbb{E}(T_{rec})$$

This is the time elapsed between the completion of the last checkpoint and the error

$$\begin{aligned}\mathbb{E}(T_{lost}) &= \int_0^\infty x \mathbb{P}(X = x | X < w + C) dx \\ &= \frac{1}{\mathbb{P}(X < w + C)} \int_0^{w+C} x \lambda_e e^{-\lambda_e x} dx \\ &= \frac{1}{\lambda_e} - \frac{w + C}{e^{\lambda_e(w+C)} - 1}\end{aligned}$$

$$\mathbb{E}(T_{lost}) + \mathbb{E}(X_d) + \mathbb{E}(T_{rec})$$

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This is the time needed for error detection,  $\mathbb{E}(X_d) = \mu_d$

$$\mathbb{E}(T_{lost}) + \mathbb{E}(X_d) + \mathbb{E}(T_{rec})$$
$$\mathbb{E}(T_{rec}) = e^{-\lambda_e R} R + (1 - e^{-\lambda_e R})(\mathbb{E}(R_{lost}) + \mathbb{E}(X_d) + \mathbb{E}(T_{rec}))$$

Let us focus on the time lost due to an error:

$$\mathbb{E}(T_{lost}) + \mathbb{E}(X_d) + \mathbb{E}(T_{rec})$$

This is the time to recover from the error (there can be a fault during recovery):

$$\begin{aligned} \mathbb{E}(T_{rec}) &= e^{-\lambda_e R} R \\ &+ (1 - e^{-\lambda_e R})(\mathbb{E}(R_{lost}) + \mathbb{E}(X_d) + \mathbb{E}(T_{rec})) \end{aligned}$$

Similarly to  $\mathbb{E}(T_{lost})$ , we have:  $\mathbb{E}(R_{lost}) = \frac{1}{\lambda_e} - \frac{R}{e^{\lambda_e R} - 1}$ .

So finally,  $\mathbb{E}(T_{rec}) = (e^{\lambda_e R} - 1)(\mu_e + \mu_d)$

$$\mathbb{E}(T(w)) = e^{\lambda_e R} (\mu_e + \mu_d) (e^{\lambda_e(w+C)} - 1)$$

This is the exact solution!

Using  $n$  chunks of size  $w_i$  (with  $\sum_{i=1}^n w_i = W$ ), we have:

$$\mathbb{E}(T(W)) = K \sum_{i=1}^n (e^{\lambda_e(w_i+C)} - 1)$$

with  $K$  constant.

Independent of  $\mu_d$ !

Minimum when all the  $w_i$ 's are equal to  $w = W/n$ .



$$\mathbb{E}(T(W)) = K \sum_{i=1}^n (e^{\lambda_e(w_i+C)} - 1)$$

with  $K$  constant.

Independent of  $\mu_d$ !

Minimum when all the  $w_i$ 's are equal to  $w = W/n$ .

Optimal  $n$  can be found by differentiation

A good approximation is  $w = \sqrt{2\mu_e C}$  (Young's formula)

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Extend results when  $X_e$  follows an arbitrary distribution of  
mean  $\mu_e$

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**Waste:** fraction of time not spent for useful  
computations

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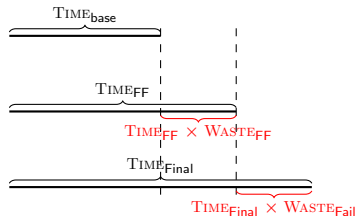
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- $\text{TIME}_{\text{base}}$ : application base time
- $\text{TIME}_{\text{FF}}$ : with periodic checkpoints but failure-free
- $\text{TIME}_{\text{Final}}$ : expectation of time with failures



$$(1 - \text{WASTE}_{\text{FF}})\text{TIME}_{\text{FF}} = \text{TIME}_{\text{base}}$$

$$(1 - \text{WASTE}_{\text{Fail}})\text{TIME}_{\text{Final}} = \text{TIME}_{\text{FF}}$$

$$\text{WASTE} = \frac{\text{TIME}_{\text{Final}} - \text{TIME}_{\text{base}}}{\text{TIME}_{\text{Final}}}$$

$$\text{WASTE} = 1 - (1 - \text{WASTE}_{\text{FF}})(1 - \text{WASTE}_{\text{Fail}})$$

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We can show that

$$\text{WASTE}_{\text{FF}} = \frac{C}{T}$$

$$\text{WASTE}_{\text{Fail}} = \frac{\frac{T}{2} + R + \mu_d}{\mu_e}$$

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Only valid if  $\frac{T}{2} + R + \mu_d \ll \mu_e$ .

$$\text{WASTE}_{\text{FF}} = \frac{C}{T}$$

$$\text{WASTE}_{\text{Fail}} = \frac{\frac{T}{2} + R + \mu_d}{\mu_e}$$

Only valid if  $\frac{T}{2} + R + \mu_d \ll \mu_e$ .

Then the waste is minimized for

$$T_{\text{opt}} = \sqrt{2(\mu_e - (R + \mu_d))C} \approx \sqrt{2\mu_e C}$$

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## Theorem

- *Best period is  $T_{opt} \approx \sqrt{2\mu_e C}$*
- *Independent of  $X_d$*



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Analytical optimal solutions, valid for arbitrary distributions, without any knowledge on  $X_d$  except its mean

However, if  $X_d$  can be arbitrary large:

- Do not know how far to roll back in time
- Need to store all checkpoints taken during execution

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Assume that we can only save the last  $k$  checkpoints

## Definition (Critical failure)

Error detected when all checkpoints contain corrupted data.

Happens with probability  $\mathbb{P}_{\text{risk}}$  during whole execution.

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$\mathbb{P}_{\text{risk}}$  decreases when  $T$  increases (when  $X_d$  is fixed).  
Hence,  $\mathbb{P}_{\text{risk}} \leq \varepsilon$  leads to a lower bound  $T_{\min}$  on  $T$

We have derived an analytical form for  $\mathbb{P}_{\text{risk}}$  when  $X_d$  follows an Exponential law. We use it as a good(?) approximation for arbitrary laws

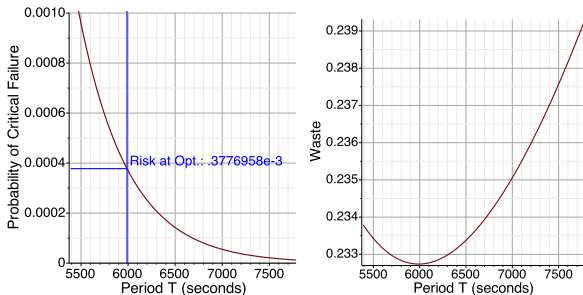


Figure :  $k = 3$ ,  $\lambda_e = \frac{10^5}{100y}$ ,  $\lambda_d = 30\lambda_e$ ,  $w = 10d$ ,  $C = R = 600s$

$T_{opt} \approx 100min$ ,  $\mathbb{P}_{risk}(T_{opt}) \approx 38 \cdot 10^{-5}$ , for a waste of 23.45%

To reduce  $\mathbb{P}_{risk}$  to  $10^{-4}$ , a  $T_{min}$  of 8000 seconds is sufficient, increasing the waste by only 0.6%. In this case, the benefit of fixing the period to  $\max(T_{opt}, T_{min})$  is obvious

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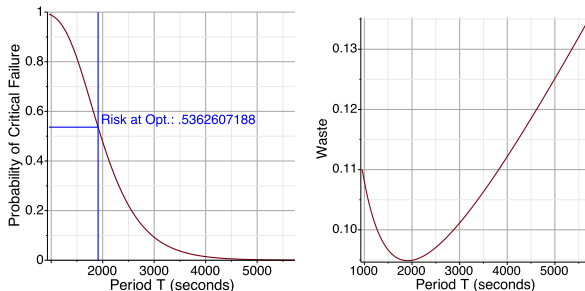
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**Figure :**  $k = 3$ ,  $\lambda_e = \frac{10^5}{100v}$ ,  $\lambda_d = 30\lambda_e$ ,  $w = 10d$ ,  $C = R = 60s$ .

## Y. Robert

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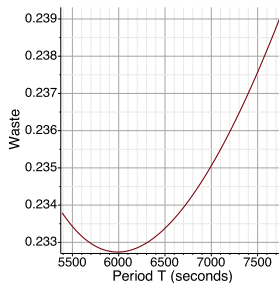
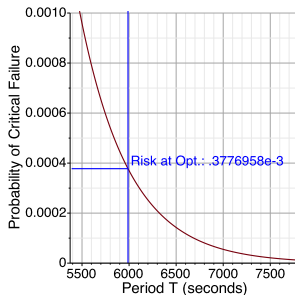
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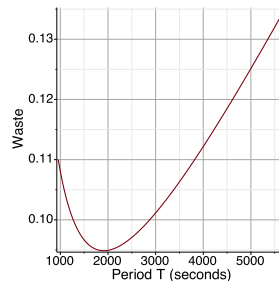
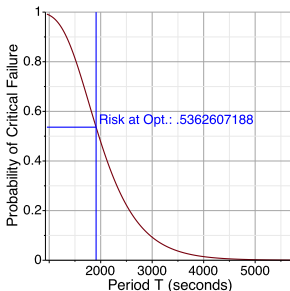
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**Figure :**  $k = 3, \lambda_e = \frac{10^5}{100v}, \lambda_d = 30\lambda_e, w = 10d, C = R = 600s$



**Figure :**  $k = 3$ ,  $\lambda_e = \frac{10^5}{100v}$ ,  $\lambda_d = 30\lambda_e$ ,  $w = 10d$ ,  $C = R = 60s$ .

It is not clear how can one detect when the error occurred  
(hence to identify the last valid checkpoint)

Need a verification mechanism to check the correctness of the checkpoints. This has a cost!

Possible solution: add verifications; use a periodic mechanism to verify that there were no silent errors in previous computations.



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Assume there are no errors during checkpoints (less error sources when doing I/O)

Simple approach: Perform a verification before each checkpoint to eliminate risk of corrupted data.

$$\text{WASTE}_{\text{FF}} = \frac{V+C}{w+V+C}, \text{WASTE}_{\text{Fail}} = \frac{w}{\mu_e}$$

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When  $V$  is large compared to  $w$ ,  $\text{WASTE}_{\text{FF}}$  is large, can we improve that?

$$\text{WASTE}_{\text{FF}} = \frac{V+C}{w+V+C}, \text{WASTE}_{\text{Fail}} = \frac{w}{\mu_e}$$

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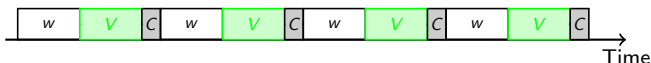
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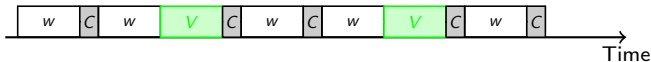
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When  $V$  is large compared to  $w$ ,  $\text{WASTE}_{\text{FF}}$  is large, can we improve that?

Is this better?



$$\text{WASTE}_{\text{FF}} = \frac{V+C}{w+V+C}, \text{WASTE}_{\text{Fail}} = \frac{w}{\mu_e}$$

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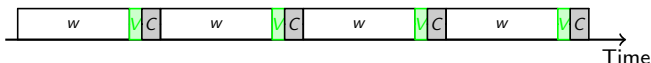
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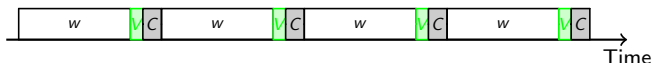


When  $V$  is small in front of  $w$ ,  $\text{WASTE}_{\text{Fail}}$  is large, can we improve that?

# Motivational Examples

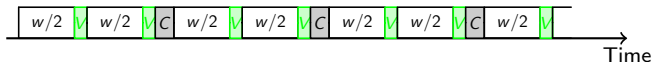
$R = 0$ :

$$\text{WASTE}_{\text{FF}} = \frac{V+C}{w+V+C}, \text{WASTE}_{\text{Fail}} = \frac{w}{\mu_e}$$



When  $V$  is small in front of  $w$ ,  $\text{WASTE}_{\text{Fail}}$  is large, can we improve that?

Is this better?



# $k$ checkpoints for 1 verification



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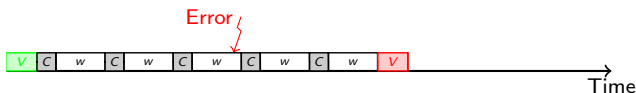
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With multiple checkpoints, the problem is to find when the error occurred.



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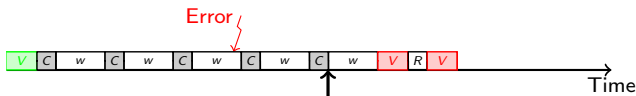
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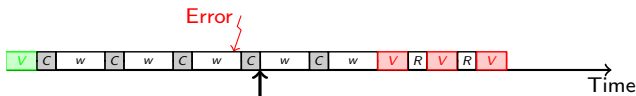
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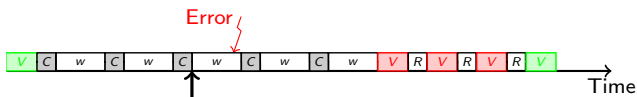
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With multiple checkpoints, the problem is to find when the error occurred.



# $k$ checkpoints for 1 verification



## Introduction

### Optimal Checkpointing strategy

Exponential  
distribution  
Arbitrary  
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### Limited resources

### Incorporating detection

$k$  checkpoints  
for 1 verification  
 $k$  verifications  
for 1 checkpoint

### Conclusion, future work

$$\text{WASTE}_{\text{FF}} = \frac{kC + V}{k(w + C) + V}$$

$$\text{WASTE}_{\text{Fail}} = \frac{\frac{1}{k} \sum_{i=1}^k T_{\text{lost}}(i)}{\mu_e}$$

where  $T_{\text{lost}}(i)$  is the time lost if error occurred in  $i^{\text{th}}$  segment

# $k$ checkpoints for 1 verification



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### Optimal Checkpointing strategy

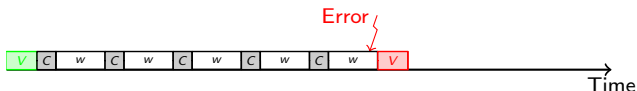
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$$T_{lost}(k) = R + V + w + V$$

# $k$ checkpoints for 1 verification



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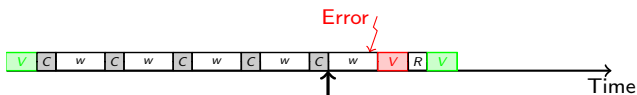
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# $k$ checkpoints for 1 verification



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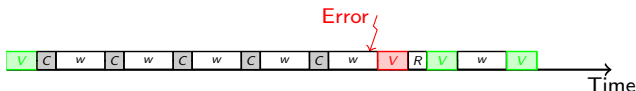
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$$T_{lost}(k) = R + V + w + V$$

# $k$ checkpoints for 1 verification



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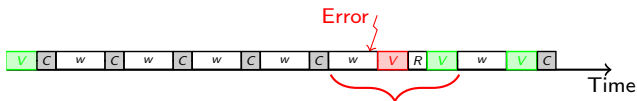
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$$T_{lost}(k) = R + V + w + V$$

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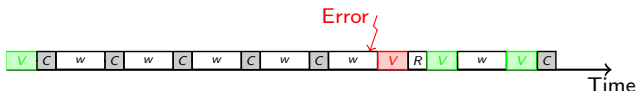
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$$T_{lost}(k) = R + V + w + V$$

$$T_{lost}(i) = (k - i + 1)(R + V + w) + (k - i)C + V$$



# $k$ checkpoints for 1 verification



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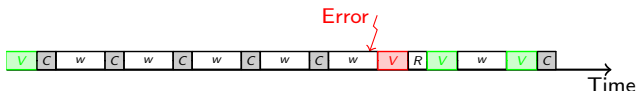
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$$T_{lost}(k) = R + V + w + V$$

$$T_{lost}(i) = (k - i + 1)(R + V + w) + (k - i)C + V$$

$$T_{lost}(1) = k(R + V + w) - V + (k - 1)C + V$$

# $k$ checkpoints for 1 verification



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## Optimal Checkpointing strategy

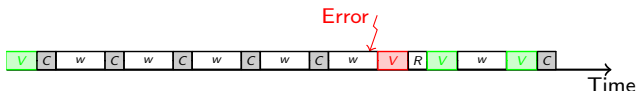
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$$T_{lost}(k) = R + V + w + V$$

$$T_{lost}(i) = (k - i + 1)(R + V + w) + (k - i)C + V$$

$$T_{lost}(1) = k(R + V + w) - V + (k - 1)C + V$$

And this leads us to optimal solution ...

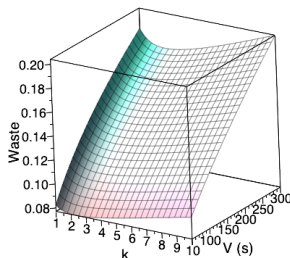
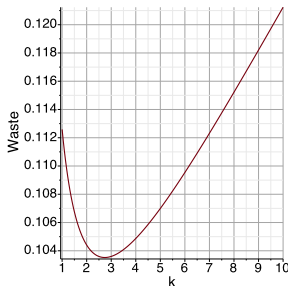
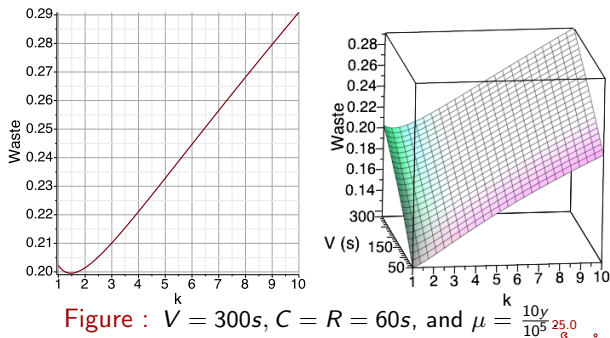


Figure :  $V = 100s$ ,  $C = R = 6s$ , and  $\mu = \frac{10\gamma}{10^5}$ .

$$C = 6s \ll V.$$

When  $V = 100$  seconds, a verification is done only every  $k = 3$  checkpoints optimally  $\Rightarrow$  10% improvement compared to  $k = 1$ .

$C = 60s$  is not negligible anymore in front of  $V$  ( $V \approx 5C$ ).  
The waste is dominated by the cost of verification, and little improvement can be achieved by taking the optimal value for  $k$ .



Silent error  
detection

Y. Robert

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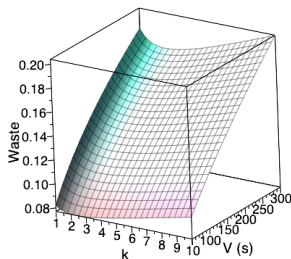
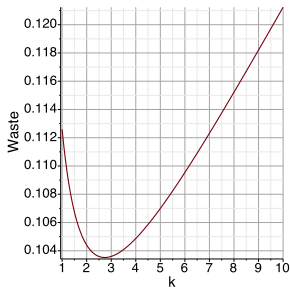


Figure :  $V = 100s$ ,  $C = R = 6s$ , and  $\mu = \frac{10y}{10^5}$ .

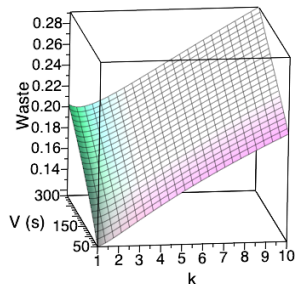
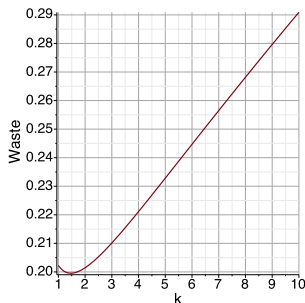


Figure :  $V = 300s$ ,  $C = R = 60s$ , and  $\mu = \frac{10y}{10^5}$   $\frac{25.0}{g}$  . . . . .



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Very similarly, we obtain:

$$\text{WASTE}_{\text{FF}} = \frac{kV + C}{k(w + V) + C}$$

$$\text{WASTE}_{\text{Fail}} = \frac{\frac{1}{k} \sum_{i=1}^k T_{\text{lost}}(i)}{\mu_e}$$

$$T_{\text{lost}}(i) = R + i(V + w)$$

where  $T_{\text{lost}}(i)$  is the time lost if error occurred in  $i^{\text{th}}$  segment.

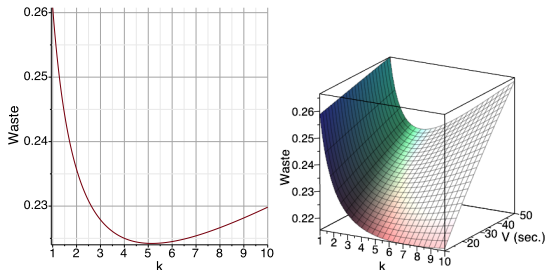


Figure :  $V = 20s$ ,  $C = R = 600s$ , and  $\mu = \frac{10y}{10^5}$ .

$$V = 20s \ll C.$$

When  $C = 600$  seconds, 5 verifications are done for every checkpoint optimally  $\Rightarrow$  14% improvement compared to  $k = 1$ .

$$V = 2s \ll C.$$

When  $C = 60$  seconds, 5 verifications are done every checkpoint optimally  $\Rightarrow$  18% improvement compared to  $k = 1$ .

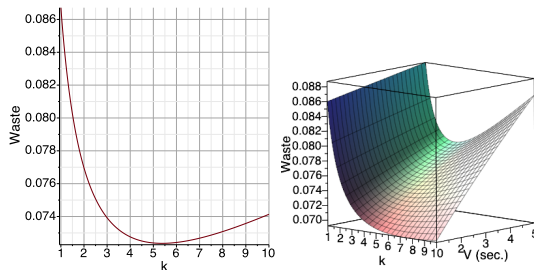


Figure :  $V = 2s$ ,  $C = R = 60s$ , and  $\mu = \frac{10\gamma}{10^5}$ .



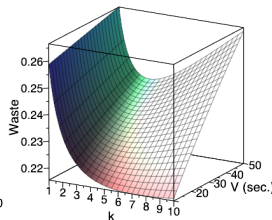
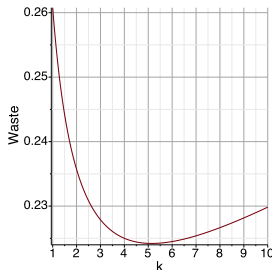


Figure :  $V = 20s$ ,  $C = R = 600s$ , and  $\mu = \frac{10y}{10^5}$ .

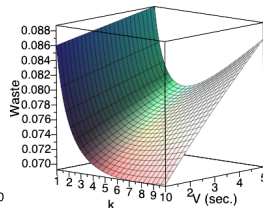
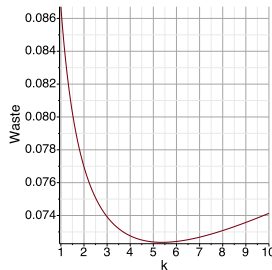


Figure :  $V = 2s$ ,  $C = R = 60s$ , and  $\mu = \frac{10y}{10^5}$ .

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## Conclusion, future work

- Study of optimal checkpointing strategy in presence of silent errors
- *Analytical* solution for the different probability distributions
- Study in presence of verification mechanisms

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- *Without verification:* When we keep  $k$  checkpoints in memory, we do not have to keep the  $k$  last checkpoints: new strategies (Fibonacci, binary, ...)?
- *With verification:* We focused on an integer number of checkpoints per verification (or conversely): extensions?